<u>Frege: Question 1</u>
Wylie Breckenridge
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“A logically perfect language … should satisfy the [condition], that every expression grammatically well constructed as a proper name out of signs already introduced shall in fact designate an object.” (FREGE, On Sense and Reference*). Discuss.*

I think there are two ways that we might understand this passage, because there are two ways that we might understand ‘already introduced’. Immediately following, Frege adds, “… and … no new sign shall be introduced as a proper name without being secured a *Bedeutung*.”[1] This might be understood either as the stipulation that no new sign *counts* as having been introduced unless it has been secured a *Bedeutung*, or as the additional requirement that for language to be logically perfect any sign that is introduced will not only have been introduced, but will *also* have been secured a *Bedeutung* (thus suggesting that signs can be introduced without having been secured a *Bedeutung*). When understood in the first way, I take Frege’s claim to be this: that in a logically perfect language every complex proper name whose constituents have a *Bedeutung* also has a *Bedeutung*. When understood in the second way, I take it to be this: that in a logically perfect language every complex proper name has a *Bedeutung*.[2]

Both claims are interesting, and both are discussed by Frege (at least implicitly) in his work. But the broader context of the passage suggests to me that we should understand him in the second way, and so I will. Since among complex proper names Frege includes both complex singular terms and sentences, the claim factors into two: (i) that in a logically perfect language every complex singular term has a denotation, and (ii) that in a logically perfect language every sentence has a truth value. I will set aside (ii) and consider just (i), firstly because I think that it is also suggested by the broader context that Frege is more concerned there with complex singular terms than with sentences, and secondly because considering just (i) will allow me to do so in more detail. In any case, my conclusion about the claim’s first factor will transmit to the claim as a whole. I will use the term ‘empty complex singular term’ to mean a complex singular term without denotation, and abbreviate it to ‘ECST’. If we take any feature of a language that detracts from its logical perfection to be a logical defect of the language, then I think we can put the claim this way:

(Claim) It is a logical defect of a language for it to contain ECSTs.

To anyone with a modern conception of logic, this might seem a strikingly odd claim. For we seem perfectly able to ‘do logic’ in our language, even though it contains ECSTs, and even using sentences that have ECSTs as constituents. If I read that the mad hatter served tea and biscuits, then I might reason to the conclusion that the mad served biscuits, even while denying that ‘the mad hatter’ has a denotation.[3] Or, to take a non-fictional case, if I were to hear you claim that the present king of France has no ears but can still hear, then, even though I believe that ‘The present king of France’ is an ECST, I might object to your claim, arguing that if the present king of France can hear then it follows that the present king of France must have ears. Sometimes not only is it possible to do logic using ECSTs, but it is very useful to do so. There is in mathematics, for example, an elegant proof that there is no greatest prime number, which starts by assuming that ‘the greatest prime number’ has a denotation, and then proceeds to derive a contradiction. The proof is an informative piece of logical argument, even though it

[1] (1997c), p. 163.
[2] I shall use ‘complex proper name’ to mean an expression grammatically well constructed as a proper name.
[3] It might be that ‘the mad hatter’ denotes a fictional character, in which case I am wrong to think that it has no denotation. Even so, that does not effect my point: I *believe* that I can do logic perfectly well with ECSTs.

uses sentences that have ‘the greatest prime number’ as a constituent, which is shown by the proof to be an ECST.[4]

But to understand Frege’s claim correctly we must understand ‘logic’ as he did. For Frege, the role of a logical language is to provide a means of deriving logical truths from logical truths by applying rules of inference (ultimately, from a finite stock of axiomatic logical laws). Note: its role is to provide a means of deriving *truths* from *truths*, not to provide a means by which we can investigate the consistency of thoughts (we might say: propositions), regardless of their truth value or whether or not they have one. If we keep this in mind, and if we also keep in mind that Frege held:

> (Thesis) *Any* sentence which has an ECST as a constituent has no truth value;

then, as will become apparent, Frege’s claim does not so immediately strike as odd.

My aim is to consider whether or not, in his many negative remarks about ECSTs, Frege gives us reason to believe Claim. I will argue that while he gives us reason to think that ECSTs are both logically *inert* and logically *dangerous* (in senses to be explained), he gives us no reason to think that a language containing them is logically *defective*. Perhaps, ultimately, this shows that the more charitable way to understand the passage in question is the *other* of the two ways that I distinguished.

I

There are many parts of his work in which Frege makes negative comments about ECSTs, or at least seems to do so. I think that we can group these comments into at least two kinds. The first includes those in the following passages. In ‘Letter to Husserl, 24.5.1891’:

> In literary use it is sufficient if everything has a sense; in scientific use there must also be *Bedeutungen*. (150)

In ‘Comments on *Sinn* and *Bedeutung*’:

> [The intensionalist logicians] forget that logic is not concerned with how thoughts, regardless of truth-value, follow from thoughts, that the step from thought to truth-value - more generally, the step from sense to *Bedeutung* - has to be taken. They forget that the laws of logic are first and foremost laws in the realm of *Bedeutungen* and only relate indirectly to sense. If it is a question of the truth of something - and truth is the goal of logic - we also have to inquire after *Bedeutungen*; we have to throw aside proper names that do not designate or name an object … [F]or fiction the sense is enough. The thought, though it is devoid of *Bedeutung*, of truth-value, is enough, but not for science. (178)

In ‘Logic’:

> Assertions in fiction are not to be taken seriously: they are only mock assertions. Even the thoughts are not to be taken seriously as in the sciences: they are only mock thoughts. … [A] work of fiction is not meant to be taken seriously … at all: it’s all play. Even the proper names in [*Don Carlos*], though they correspond to names of historical personages, are mock proper names; they are not meant to be taken seriously in he work. … The logician does not have to bother with mock thoughts, just as a physicist, who sets out to investigate thunder, will not pay any attention to stage-thunder. When we speak of thoughts … we mean thoughts proper, thoughts that are either true or false. (130)

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4 It might be possible to word the proof so that it does not use the expression ‘the greatest prime number’. Even so, it is usual to use this expression, because it seems most convenient to do so.

In ‘Grundgesetze der Arithmetik, Volume II’:

> [The sentence ‘Scylla had six dragon necks’] is neither true nor false, but fiction, for the proper name ‘Scylla’ designates nothing. Such sentences can indeed be objects for scientific treatment, e.g. of myth; but no scientific investigation can issue in them. (266)

In ‘A brief Survey of my logical Doctrines’:

> In myth and fiction thoughts occur that are neither true nor false. Logic has nothing to do with these. In logic it holds good that every thought is either true or false, *tertium non datur*. (198)

And in ‘Letter to Jourdain, Jan. 1914’:

> Without a *Bedeutung*, we could indeed have a thought, but only a mythological or literary thought, not a thought that could further scientific knowledge. (320)

Clearly, Frege makes a distinction between expressions that have a sense but no *Bedeutung*, calling them ‘fictitious’ or ‘not serious’ or ‘mock’, and expressions that have both a sense and a *Bedeutung*, calling them ‘scientific’ or ‘serious’. And clearly, he thinks that only the second kind of expression is of interest to science and hence logic (logic itself being a science, ‘of the most general laws of truth’).[5] I take it that this is why: The task of a logical language is to provide a means of expressing logical truths, and of deriving logical truths from logical truths by the application of rules of inference; logical truths are expressed by sentences, and derivations of logical truths from logical truths are sequences of such sentences; each of these sentences expresses a true thought, so each has a *Bedeutung*, and every constituent of each has a *Bedeutung* (otherwise, by Thesis, the sentence would not have a *Bedeutung*); so expressions without *Bedeutung* and, in particular, ECSTs, cannot appear in any of these sentences; so they are of no concern to logic.

If this is what Frege has in mind, then I think that he has given us a reason to think that ECSTs are what we might call logically *inert* **-** that they can play no role in a logical language. But this, by itself, is no reason to think that a language containing them is logically defective, for it is no reason to think that they in any way *hamper* the expression and derivation of logical truths. It shows that we cannot use ECSTs to do logic, but it does not show that they in any way prevent us from doing logic.

II

A second kind of negative comment that Frege makes about ECSTs has to do with his concern that they can lead us into error. Examples can be found in the following passages. In ‘Function and Concept’:

> It seems to be demanded by scientific rigour that we ensure that an expression never becomes *bedeutungslos*; we must see to it that we never perform calculations with empty signs in the belief that we are dealing with objects. People have in the past carried out invalid procedures with divergent infinite series. (141)

In ‘On *Sinn* and *Bedeutung*’:

> The logic books contain warnings against logical mistakes arising from the ambiguity of expressions. I regard as no less pertinent a warning against apparent proper names that

[5] (1979a), p. 128.

have no *Bedeutung*. The history of mathematics supplies errors which have arisen in this way. This lends itself to demagogic abuse as easily as ambiguity - perhaps more easily…. It is therefore by no means unimportant to eliminate the source of these mistakes, at least in science, once and for all. (163-4)

In 'Comments on *Sinn* and *Bedeutung*':

> [I]n fiction words only have a sense, but in science and wherever we are concerned about truth, we are not prepared to rest content with the sense, we also attach a *Bedeutung* to proper names and concept words; and if through some oversight, say, we fail to do this, then we are making a mistake that can easily vitiate our thinking. (173)

And:

> In my *Grundlagen* and the paper 'On Formal Theories of Arithmetic' I showed that for certain proofs it is far from being a matter of indifference whether a combination of signs - e.g. √-1 - has a *Bedeutung* or not, that, on the contrary, the whole cogency of the proof stands or falls with this. (178)

And, in 'Sources of Knowledge of Mathematics and the mathematical natural Sciences':

> One feature of language that threatens to undermine the reliability of thinking is its tendency to form proper names to which no objects correspond. If this happens in fiction, which everyone understands to be fiction, this has no detrimental effect. It's different if it happens in a statement which makes the claim to be strictly scientific. A particularly noteworthy example of this is the formation of a proper name after the pattern of 'the extension of the concept *a*', e.g. 'the extension of the concept *star*'. Because of the definite article, this expression appears to designate an object; but there is no object for which this phrase could be a linguistically appropriate designation. From this have arisen the paradoxes of set theory which have dealt the death blow to set theory itself. … It is difficult to avoid an expression that has universal currency, before you learn of the mistakes it can give rise to. (269)

What kinds of error is Frege concerned about? In 'On *Sinn* and *Bedeutung*', he describes one to do with the sentence, 'Whoever discovered the elliptic form of the planetary orbits died in misery,' and its subordinate clause, 'Whoever discovered the elliptic form of the planetary orbits.'[6] By Thesis, the thought that the sentence expresses cannot be true unless the subordinate clause has a *Bedeutung*. This might lead someone to think (mistakenly, Frege took it) that the thought contains as a constituent the further thought that there was someone who discovered the elliptic form of the planetary orbits, so that to assert the sentence is to assert *both* that there was someone who discovered the elliptic form of the planetary orbits, *and* that whoever discovered the elliptic form of the planetary orbits died in misery. Such a mistake is made possible by the fact that there are in our language ECSTs, and we realise that there are. If there were no such terms, and we knew that there were no such terms, then we would not be tempted to think that by asserting that whoever discovered the elliptic form of the planetary orbits died in misery, one is, in part, asserting that there was someone who discovered the elliptic form of the planetary orbits - there would simply be no need to do so.

Frege gives two more examples in 'Sources of Knowledge of Mathematics and the mathematical natural Sciences'.[7] The first has to do with expressions of the form, 'the extension of the concept *a*', and the second has to do with expressions of the form 'the concept *a*'. In each case Frege warns against the problems that can arise from overlooking that there can be

[6] (1997c), pp. 162-4.
[7] (1979d), pp. 269-70.

complex proper names that have no denotation. For suppose we think that if '*a*' stands for a concept then the expression 'the extension of the concept *a*' denotes an object. Then we might think that the expression, 'the extension of the concept *extension of a concept under which it does not fall*', denotes an object. But then we will get into a tangle over the question, 'Does this object fall under the concept *extension of a concept under which it does not fall*?' If we say that it does, then it seems that we must also say that it does not, and if we say that it does not, then it seems that we must also say that it does. Taking there to be an object denoted by 'the extension of the concept *extension of a concept under which it does not fall*' seems to commit us to a contradiction. And suppose we think that if '*a*' stands for a concept then the expression 'the concept *a*' has a denotation. Then we might think that the expression 'the concept *star*' has a denotation. But then we seem to have to say that it denotes an object, because it is a singular term, and that it denotes a concept, because it denotes the <u>concept</u> *star*. But, on Frege's understanding, if it denotes an object then it cannot denote a concept. Frege says, 'The difficulties which this idiosyncrasy of language entangles us in are incalculable'[8] (and he goes to great length in 'On Concept and Object' to try to untangle it). Such mistakes are made possible by the fact that there are in our language ECSTs, and we sometimes overlook that fact.[9]

If this is what Frege has in mind, then I think that he has given us a reason to think that ECSTs are what we might call logically *dangerous* - that if we are not careful they can lead us into contradiction. But this, by itself, is no reason to think that a language containing them is logically defective, for it is no reason to think that if we are careful then they in any way hamper the expression and derivation of logical truths.

III

It is tempting to think that Frege makes a third kind of negative comment about ECSTs, related to what he says about the need for concepts to be completely defined. In 'Function and Concept', he says:

> … if [the requirement that concepts have sharp delimitation] were not satisfied it would be impossible to set forth logical laws about [concepts]. (141)

In 'Grundgesetze der Arithmetik, Volume II':

> [Concepts without sharp boundaries] cannot be recognized as concepts by logic; it is impossible to lay down precise laws for them. (259)

And:

> … [The laws of logic] certainly presuppose that concepts, and relations too, have sharp boundaries. (265)

And:

> Here again we likewise see that the laws of logic presuppose concepts with sharp boundaries, and therefore also complete definitions for names of functions… (268)

In these passages, Frege is claiming that there can be no logical laws for concepts unless concepts have sharp boundaries - unless, that is, every concept is such that for every object it is either true or false that the object falls under that concept. I take it that his reason is something

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8 (1979d), p. 270.

9 It is not entirely clear whether or not we should think of 'the concept *star*' as a complex singular term, because it is not entirely clear whether or not this is a meaningful use of 'concept'. If not, then this example is not to the point.

like the following. Take an example of what we would like to be a logical law for concepts (written in modern notation):

(1) $(a = b) \rightarrow (Fa \rightarrow Fb)$

For Frege, (1) expresses a truth - it is not a sentence schema each of whose instances expresses a truth. So he would understand '*a*' and '*b*' to be variables that range over objects, '*F*' to be a variable that ranges over concepts, and for (1) to involve implicit quantification over '*a*', '*b*' and '*F*' (we might, then, write it more fully as $(\forall a)(\forall b)(\forall F)[(a = b) \rightarrow (Fa \rightarrow Fb)]$). Frege takes all quantification to be unrestricted, so that '*a*' and '*b*' range over *all* objects, and '*F*' ranges over *all* concepts. Now, if G is a concept that does not have sharp boundaries (to use Frege's term), then there is an object x such that it is neither true nor false that Gx, and so it is neither true nor false that (x = x) → (Gx → Gx) (because, by Thesis, if 'Gx' lacks a *Bedeutung* then so does '(x = x) → (Gx → Gx)'). But if it is neither true nor false that (x = x) → (Gx → Gx) then it is not true that (x = x) → (Gx → Gx), and so (1) is not true because it has an instance that is not true. (1) cannot be a law of logic unless all concepts have sharp boundaries. Generalising the argument would establish that there cannot be *any* logical laws about concepts unless all concepts have sharp boundaries.[10]

Frege argues that a similar result holds for (first level) functions (of one variable): that there cannot be any logical laws about concepts unless all functions take values on all objects.[11] Here is one way to argue the point (similar to, but not the same as the argument given by Frege). Suppose that f is a function that does not take a value on the object x (so that there is no such object as f(x)). Suppose also that f takes a value on the object y (so that there is such an object as f(y)).[12] Since there is an object f(y), the expression 'f(y) = f(y)' has a *Bedeutung* (it is the True), and so the incomplete expression 'f( ) = f(y)' has a *Bedeutung* too (for if it did not then neither would 'f(y) = f(y)'). That is, there is a concept for which the expression 'f( ) = f(y)' stands (i.e. that is the *Bedeutung* of the expression). Now, since there is no such object as f(x), the expression 'f(x)' lacks a *Bedeutung*, and hence, by Thesis, so does the expression 'f(x) = f(y)'. But that is to say that it is neither true nor false that x falls under the concept that 'f( ) = f(y)' stands for, and so the concept that 'f( ) = f(y)' stands for is a concept without sharp boundaries. So there is a concept without sharp boundaries, and by the previous result there cannot be any logical laws about concepts.

One might be tempted to read into Frege at this point an argument to the effect that at least some ECSTs are logically defective. Consider the ECST, 'The present king of France', for example. Since it has no denotation, the argument might go, it follows that the function that 'The present king of ( )' stands for does not take a value on France. But this function does take a value on Jordan, so we can apply the result of the previous paragraph and conclude that there is at least one concept without sharp boundaries, and hence that there cannot be any logical laws about concepts. So because the complex singular term 'The present king of France' has no denotation, there can be no logical laws about concepts. If this argument is sound, then it would indeed suggest that it is a logical defect of a language for it to contain this particular ECST. And if a similar argument can be run for any ECST, then that would suggest that the presence of *any* such term in language is a logical defect.

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[10] I'm not sure of the extent to which the argument will generalise. It can certainly be extended to cover any law that contains, like (1), a term '*Fa*' where '*F*' is a free variable ranging over concepts and '*a*' is a free variable ranging over objects. I will assume that it can be extended to cover any law, but nothing will turn on that assumption.

[11] (1997f), p. 265.

[12] This supposition is crucial to the argument, to guarantee that there is a concept that 'f( ) = f(y)' stands for. So the claim that it establishes is really this: that there cannot be any logical laws about concepts unless every function that takes a value on one object takes a value on every object. This weaker claim shall be strong enough for my purposes.

But even if such arguments can be run, this is not the conclusion that Frege would draw from them. What the argument in the previous paragraph shows is that these two claims are inconsistent: (1) that 'The present king of ( )' stands for a function, and (2) that every function takes a value on every object. It took (1) as a premise and concluded that (2) is false, and thereby that there can be no logical laws about concepts. But Frege takes it that there *can* be logical laws about concepts, and thus that (2) must be true. He would, then, take (2) as a premise and conclude that (1) is false. This is why he insists that we have not succeeded in giving a function word 'f' a *Bedeutung* unless we have specified a *Bedeutung* for 'f(a)' for every proper name 'a' that has a *Bedeutung*. And it is for similar reasons that he insists that we have not succeeded in giving a concept word 'F' a *Bedeutung* unless we have specified a *Bedeutung* for 'Fa' for every proper name 'a' that has a *Bedeutung*.

In his comments about the need for complete definitions of concepts and functions, then, we should not think that Frege is arguing that if there are ECSTs in a language then we cannot formulate (at least some) logical laws in that language, which would indeed be a logical defect of that language. Rather, he is arguing that it follows from the laws of logic that if a complex singular term has no denotation then at least one of its constituents has no denotation (*Bedeutung*) (i.e. the contrapositive of the *other* of the two claims that I distinguished at the start).

There is, then, no argument to be found here in support of Claim. And I can find no other arguments in Frege's writing. I conclude, then, that while Frege has given us reason to think that ECSTs are logically *inert* and logically *dangerous*, he has given us no reason to think that a language containing them is logically defective.

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