

Frege on the Reference of Incomplete Expressions

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Frege claimed the existence of things he called *functions*, and distinguished them from objects by their inability to be referred to by any of the expressions that we use to refer to objects. I want to argue that if there are such things, Frege cannot say that there are nor indeed say anything about them, and that therefore they cannot play the role that he wants them to play in his philosophy of language.

Among the expressions of a language, Frege drew a distinction between those that are *complete* and those that are *incomplete*. In English, the first type includes proper names like 'Socrates', definite descriptions like 'the greatest philosopher', and sentences like 'Socrates is a philosopher'. The second type includes predicates like '() is a philosopher', and relational expressions like '() is married to []'. Incomplete expressions can be thought of what is left behind when one or more complete expressions are removed from a sentence: '() is a philosopher' is left behind when we remove 'Socrates' from 'Socrates is a philosopher', and '() is married to []' is left behind when we remove 'Socrates' and 'Xanthippe' from 'Socrates is married to Xanthippe'. The parentheses '()' and '[]' in a name for an incomplete expression are not to be thought of as standing for parts of that expression, but for places in which the expression is incomplete - merely indicating the place(s) in which complete expressions have been removed from a sentence in order to obtain the incomplete expression, or, equally well, as indicating the place(s) in which complete expressions must be inserted into the incomplete expression in order to obtain a sentence (with different styles of parentheses being used to indicate that different complete expressions may be inserted in each place).¹

According to Frege, if a complete expression is a component of a sentence that has a truth-value, then it contributes to the truth-value of the sentence in virtue of having a *reference*. For it to have a reference is for it to stand in a certain relation to an object, and when it does its reference, or that to which it refers, is that object, and what it contributes to the truth-value of a sentence, its *semantic value*, is the object to which it refers. Frege thinks of a complete expression as purporting to name an object, of its having a reference as actually naming an object, of its reference as being the object that it names, and of its reference as being that which it contributes to the truth-value of a sentence. 'Socrates' names Socrates, so has Socrates as its reference and contributes Socrates to the truth-value of a sentence; 'Socrates is a philosopher' names the True, so has the True as its reference and contributes the True to the truth-value of a sentence; but 'the present King of France' fails to name, so does not have a reference and contributes nothing to the truth-value of a sentence.

Frege makes similar claims about *incomplete* expressions. If an incomplete expression is a component of a sentence that has a truth-value, then it contributes to the truth-value of the sentence in virtue of having a reference. For it to have a reference is for it to stand in a certain relation, not to an object but to a function, and when it does its reference, or that to which it refers, is that function. But a function is not an object. Like the expressions whose reference they might be functions are incomplete, and like the expressions whose reference they might be objects are complete, and nothing can be

¹ Here I am following Furth ??? in using parentheses to mark the 'gaps' in an incomplete expression, rather than following Frege in using Greek letters.

both complete and incomplete. Thus, crucially, functions cannot be referred to by complete expressions. Furthermore, what an incomplete expression contributes to the truth-value of a sentence, its semantic value, is not the *function* to which it refers, but the *object* that is named when it is completed. So, the contribution that '() is a philosopher' makes to the truth value of 'Socrates is a philosopher' is not the function to which '() is a philosopher' refers, but the object named when it is completed by 'Socrates' to obtain the complete expression 'Socrates is a philosopher' (in this case, the True).

If there are functions of the kind that Frege claims, then he cannot say so. Suppose he were to claim, 'there are functions'. The logical form of such a statement is standardly taken to be

$$(1) \quad (\exists x)(x \text{ is a function}).$$

Instances of (1) are obtained by replacing the 'x' inside 'x is a function' with an expression. If this expression is complete then, according to Frege, it can only refer to an object, and it is false to predicate of any object that it is a function. On the other hand, if the expression is incomplete then the result is not a sentence. So no instance of (1) is a true sentence, and that makes it hard to see why we should take it to express the truth that Frege claims it does.

Even if Frege cannot make a general claim about the existence of functions, perhaps he can still claim of some particular incomplete expression that there is a function to which it refers. No. Suppose he wanted to claim that the predicate '() is a philosopher' refers to a function. If we take the logical form of this statement to be

$$(2) \quad Rp,$$

and understand it as predicating the property, R, of referring to a function, to the predicate, p, '() is a philosopher', then this is not to understand it as saying that there is a function to which the predicate refers, but merely as saying that the predicate has a certain property. But if we take its logical form to be

$$(3) \quad (\exists x)((x \text{ is a function}) \ \& \ (\text{the predicate '() is a philosopher' refers to } x))$$

Then we encounter the same problem that we encountered with (1). Instances of (3) are obtained by replacing occurrences of 'x' in the quantified expression with either complete or incomplete expressions. In the first case the first conjunct is false, and in the second case the result is not a sentence. So, again, no instance of (3) can be a true sentence, and that makes it hard to see why we should accept it as true.

Frege offers another way of saying that an incomplete expression refers to a function. He says that '() is a philosopher', for example, refers to a function just in case every completion of the predicate by a complete expression that refers to an object is a complete expression that refers to an object. Following Furth ????, we might write the logical form of this statement as

$$(4) \quad (a)(x)(a \text{ DEN } x \rightarrow (\exists y)(C(' () \text{ is a philosopher}', a) \text{ DEN } y)),$$

where the variable 'a' ranges over complete expressions, the variables 'x' and 'y' range over objects, the relation 'DEN' is the relation that obtains between a complete expression that refers and the object to which it refers, and 'C' is a binary function whose first argument is a predicate, whose second argument is a complete expression, and whose value is the complete expression obtained by completing the first argument with the second. I can accept this as saying that the predicate '() is a philosopher' has a certain property, and I can accept, at a pinch, calling this property 'referring to a function'. But I cannot accept it as saying, literally, that the predicate refers to a function - that is, that it stands in some relation to some function. Which function is that? Which parts of (4) are supposed to *stand for* that function?

So Frege cannot truly claim that there are functions, nor that a particular incomplete expression refers to a function. Let's grant, nevertheless, that there is a function that the predicate '() is a philosopher' refers to, even though we cannot say that there is. (Frege would call this particular type of function a *concept*, because it is the reference of a predicate. I will do the same.) Can Frege say anything about this concept? We cannot refer to it using complete expressions like "the concept that '() is a philosopher' refers to", or "the reference of '() is a philosopher'", because complete expressions cannot refer to concepts. If these refer to anything at all then they refer to objects, not concepts, so if we try to use them to talk about the concept that we want to we end up, at best, talking about an object instead. We might think that we can use the following sentences to say of the concept in question that it is a concept:

- (5) The reference of '() is a philosopher' is a concept;
- (6) The concept that '() is a philosopher' refers to is a concept.

But neither of these can be true. In each case we are trying to say of something that it is a concept, but because we are trying to refer to that thing using a complete expression then if we do successfully refer it can only be to an object, and then we cannot truly say of it that it is a concept. What we *might* be able to truly say is this:

- (7) The concept that '() is a philosopher' refers to is not a concept.

If "The concept that '() is a philosopher' refers to" refers to anything then, because it is a complete expression, it refers to an object, and in that case it is true to say of that thing that it is not a concept. But even though (7) may be a true statement, it is not a statement about a *concept*, let alone the concept that we are trying to talk about.

We cannot say anything about the concept using incomplete expressions either. We can refer to it because, as we are temporarily granting, the predicate '() is a philosopher' refers to it. But that does not mean that we can *talk about* it. We cannot, for example, use the sentence

- (8) () is a philosopher is a concept

to say that it is a concept, simply because (8) is not a complete expression and therefore not a sentence. We might try, following Frege (CSB) and Dummett (1981), introducing the expression "what '() is a philosopher' refers to", declaring it to be an incomplete expression with the same reference as '() is a philosopher', and then saying

- (9) What '() is a philosopher' refers to is a concept.

But this does not say what we want it to say either, for the same simple reason that it is not a sentence. To think that it is is to forget that "what '() is a philosopher' refers to" is being taken as an *incomplete* expression, and not to realise that (9) is grammatically on par with (8). We *can* use the incomplete expressions '() is a philosopher' and "what '() is a philosopher' refers to" in well-formed sentences, thereby referring to the concept in question:

- (10) Socrates is a philosopher;
(11) Socrates is what '() is a philosopher' refers to.

These are both true, and each involves an expression that refers to the concept in question. But neither says anything about it. In both cases we are saying *of Socrates* that he is a philosopher. In both cases we are talking about *Socrates* and not about the concept. It might be claimed that (11) can be rewritten as "What '() is a philosopher' refers to is Socrates", which does indeed seem to be saying something about the concept. But (11) cannot be written this way - to think that it can is to mistake the 'is' of predication for some other 'is', perhaps of identity. If x is F, where 'is' is the 'is' of predication, then we cannot equivalently say that F is x. It might be said that if we are referring to the concept in (10) and (11) then we are saying something about it. But that would be to assume that, like a complete expression, the semantic contribution of an incomplete expression to a sentence is its reference, and we have seen that that is not so.

There is a sense in which (10) says something about things other than Socrates. It says of the predicate '() is a philosopher', for example, that it truly applies to Socrates, and of the extension of the predicate, for example, that it includes Socrates. But these are claims about the predicate and its extension, not about the *concept* that the predicate refers to. Can we think of (10) as saying something about that concept? Frege would like to say that it says that Socrates falls under the concept, and that in that sense it says something about the concept. That is, Frege would like to say that (10) says the same as

- (12) Socrates falls under the concept that '() is a philosopher' refers to.

But to think of this as saying that Socrates stands in the relation of 'falling under' to the concept that '() is a philosopher' refers to, is to think of it as saying something about Socrates and some *object* – that they stand in a certain relation – and not about Socrates and some *concept*. This is because the expression "the concept that '() is a philosopher' refers to" is a complete expression and so refers to an object and not a concept. On the other hand, to think of (12) as saying that Socrates has the property of falling under the concept that '() is a philosopher' refers to is to think of it as saying something about Socrates (that he has a certain property) or possibly about a certain property (that Socrates has it), and not as saying anything about a concept, let alone the concept that Frege would like to be talking about. So if Frege were to claim that (10) says the same thing as (12) he would not thereby be claiming that (10) says anything about a concept, because (12) says nothing about a concept.

If there are such things as concepts then we might expect them to stand in certain relations to each other. There are two relations in particular that Frege would like to be able to talk about. The first is the relation that two concepts stand in if and only if they

are ‘identical’. I use scare-quotes here because concepts cannot stand in the relation of *identity*: identity can only hold between objects and concepts are not objects. Frege claims that there is an analogous relation for concepts. He says that it “holds between the concept Φ and the concept X, if every object that falls under Φ also falls under X, and conversely.”² As I have already argued, we cannot sensibly take these words of Frege’s literally without taking him to be talking about objects, not concepts, and thus as making quite a different claim to the one he wants to make. If P and Q are predicates that have a reference, Frege cannot say, of any relation, that it holds between the concepts to which they refer, because we cannot make statements about those concepts using complete sentences. The closest he can come to saying what he wants to say is this: that there is a certain relation that holds between predicates with a reference such that it holds between P and Q if and only if for every complete expression E that refers to an object the object that the completion of P by E refers to is identical to the object that the completion of Q by E refers to, and that this relation should be thought of as the ‘identity’ relation for concepts. If we let ‘ \approx ’ stand for this relation then we can express this symbolically as:

$$(13) \quad (P \approx Q) \leftrightarrow (a)(x)(a \text{ DEN } x \rightarrow (y)(z)(C(P,a) \text{ DEN } y \ \& \ C(Q,a) \text{ DEN } z \rightarrow y = z)),$$

where ‘a’, ‘x’, ‘y’, ‘DEN’ and ‘C’ are as before, and ‘z’ is, like ‘x’ and ‘y’, a variable that ranges over objects.

Frege would also like to talk about a relation that obtains between two concepts when one is *subordinate* to the other. Similar to what he says about the previous relation, he might say that the concept Φ is subordinate to the concept X if and only if every object that falls under Φ also falls under X. Since all whales are mammals, he would say, then, that the concept *whale* is subordinate to the concept *mammal*. Again we cannot sensibly take these claims literally without taking them to be talking about objects, and thus as making quite different claims to the ones that Frege wants to make. The closest he can come to saying what he wants is this: there is a certain relation that holds between predicates with a reference such that it holds between P and Q if and only if, for every complete expression E that refers to an object, if the object that the completion of P by E refers to is the True, then the object that the completion of Q by E refers to is the True, and that this relation should be thought of as the subsumption relation for concepts. If we let ‘ $<$ ’ stand for this relation then we can express this symbolically as:

$$(14) \quad (P < Q) \leftrightarrow (a)(x)(a \text{ DEN } x \rightarrow (C(P,a) \text{ DEN the True} \rightarrow C(Q,a) \text{ DEN the True})).$$

If (13) and (14) are the best way of making sense of Frege’s claims about concepts standing in certain relations, then I don’t what reason we have to think that there are such things as relations that obtain between concepts. Sure, we might express (13) more simply by saying that ‘The concept that P refers to is the same as the concept that Q refers to’, and (14) more simply by saying that ‘The concept that P refers to is subordinate to the concept that Q refers to’. But I don’t see that we have any right to take these as anything more than convenient manners of speaking, with no ontological commitment to concepts and their ability to stand in relations. I don’t see how we can

² CSB p. 176.

think of (13) and (14) as expressing a relation between two concepts, if we cannot find a logical form in which it is clear that two concepts are being mentioned.

Frege is aware that it is difficult for him to say what he wants to say about functions:

I admit that there is a quite peculiar obstacle in the way of understanding with my reader. By a kind of necessity of language, my expressions, taken literally, sometimes miss my thought; I mention an object, when what I intend is a concept. I fully realize that in such cases I was relying upon a reader who would be ready to meet me halfway - who does not begrudge a pinch of salt.³

I do begrudge a pinch of salt. If what I have said is right, the problem that Frege has with saying what he wants to say about functions is not linguistic but logical. He might be able to talk *as if* there are and *as if* about them, but there is nothing to be found in the logical form of such talk to warrant us thinking that it is about anything other than objects. In many ways this does not matter: although when he says that the concept *man*, for example, is not empty we must take him to really be saying that there is a completion of '() is man' by an complete expression that refers to the True, as far as his logic is concerned it makes little difference. But Frege denies that talk as if there are functions is just a terminological convenience. He wants there to actually *be* functions to figure in metaphysical explanation. He says that "not all parts of a thought can be complete; at least one must be unsaturated or predicative; otherwise they would not hold together",⁴ and elsewhere he appeals to the brute incompleteness of concepts to stop a regress of explanation of how concepts and objects can 'bind together' in states of affairs. But if, as I have argued, Frege cannot say that there *are* such functions, let alone say anything about them, then he cannot have given an explanation from which any regress can even begin.

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³ CO p. 192.

⁴ CO p.193.

