

## What, for Frege, is logic?

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Frege said that “logic is the science of the most general laws of truth”.<sup>1</sup> My aim here is to show just how much logic as he conceives it deserves to be called a science, not in any idiosyncratic sense of the word, but in the sense in which we today say that physics and chemistry are sciences.

What is our conception of physics and chemistry? No doubt a difficult question, but a partial answer listing some key features shall serve my purpose. Both aim to make true statements about the world, particular and general. Physics claims, for instance, that the speed of sound in air is not greater than the speed of light in a vacuum, and, more generally, that nothing travels faster than the speed of light in a vacuum. Chemistry claims that Helium is an inert gas, and, more generally, that any gas with a full outer electron shell is inert. General statements, or laws, differ from particular statements only in their generality - a law can be instantiated to obtain many particular statements, and many particular statements can be subsumed under a law. That the speed of sound in a vacuum is not greater than the speed of light in a vacuum is an instance of the physical law that nothing travels faster than the speed of light in a vacuum; that Helium is an inert gas is an instance of the chemical law that any gas with a full outer electron shell is inert. Both seek laws, in part, for their explanatory value. Why is Helium an inert gas? An unsatisfying explanation would be that Helium has a full outer electron shell, and that if Helium has a full outer electron shell then it is inert. A more satisfying answer (somewhat, at least) is that Helium has a full outer electron shell and that for any gas, if it has a full outer electron shell then it is inert. The satisfaction comes from the appeal to a law. Both aim to develop theories consisting of simple, explanatorily complete systems of laws and particular statements, and employ, among other techniques, the method of trial and error to do so. Physics and chemistry can be distinguished by their subject matter - by the kinds of things the truths at which they aim are about. Physics is about speed, sound, vacuums and light; chemistry is about gases, electron shells, and inertness. But there is overlap - both are about speed and both are about electrons, and some laws of chemistry are instances of laws of physics, or can be explained by the laws of physics.

How does Frege’s conception of logic compare to the (partial) conception of physics and chemistry that I have given here? Frege says that “[t]he word ‘true’ can be used to indicate the goal for logic...”.<sup>2</sup> He notes that “all sciences have truth as their goal, but logic is concerned with the predicate ‘true’ in a quite special way, namely in a way analogous to that in which physics has to do with the predicates ‘heavy’ and ‘warm’ or chemistry with the predicates ‘acid’ and ‘alkaline’”.<sup>3</sup> He goes on to say, “There is, however, the difference that these sciences have to take into account other properties besides these we have mentioned, and that there is no one property by which their nature is so completely characterized as logic is by the word ‘true’”.<sup>4</sup> Here Frege is clearly taking logic to be a science alongside physics and chemistry, and is trying to distinguish the three. What is the special way in which logic is concerned with the predicate ‘true’? He draws an analogy with the way in which physics has to do with the

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<sup>1</sup> L2.

<sup>2</sup> L2.

<sup>3</sup> L2.

<sup>4</sup> L2.

predicates ‘heavy’ and ‘warm’. What is that analogy? It can’t be that logic studies the property of truth in the way that physics studies the property of heaviness and warmth, because elsewhere Frege casts doubt upon there being a property of truth (reference?). It can’t be that logic is about truth in the way that physics is about heaviness and warmth, because the claims that Frege takes to be logical claims are not about truth - they do not, for instance, employ the word ‘true’, and it is no part of their content that such-and-such is true. I suggest that the analogy is this: the objects that physics studies are those that can be heavy or light, warm or cold, and so on; the objects that chemistry studies are those that can be acid or alkaline, solid or liquid, and so on; similarly, the objects that logic studies are those that can be true or false, but in this case being true or false completely characterises those objects. So, if we want words with which to mark of the subject matter of each science, then in the case of physics we will need a list of words including ‘heavy’ and ‘warm’, in the case of chemistry we will need a list of words including ‘acid’ and ‘alkaline’, but in the case of logic we need just the word ‘true’.

Frege conceives of logical statements and laws as making claims about the world, just as we conceive that physics and chemistry do. Compare the general physical claim that nothing travels faster than the speed of light in a vacuum with the general logical claim, for Frege, that for all propositions (what he would call “thoughts”)  $p$  and  $q$ , if  $p$  and  $q$  then  $p$ :  $(\forall p)(\forall q)(p \& q \rightarrow p)$ . The general physical claim is quantified over all objects and phenomena in the world. The general logical claim is quantified over all propositions. For Frege, propositions were very much a part of the world, not a part of the physical world, but a part of the world nonetheless. So, in the sense that both quantify over objects in the world, the logical claim is about the world just as much as the physical claim is. In contrast, the claims of present-day model-theoretic logic are naturally understood as not being about the world. They are about sentential forms (or schemata), and so are about the language we use to make claims about the world, not about the world itself. Such a claim might be that the schema  $p \& q \rightarrow p$  is valid, by which is meant that if ‘&’ and ‘ $\rightarrow$ ’ are interpreted in their intended way, then no matter how the symbols ‘ $p$ ’ and ‘ $q$ ’ are interpreted the sentence obtained expresses a truth. Intuitively, this is not about the world but about the meanings of the symbols ‘&’ and ‘ $\rightarrow$ ’. A related point is one that I mentioned earlier. On our conception of particular and general physical claims there is no difference in kind between them - they differ only in their degree of generality. To support this, note that we can think of asserting a general claim as asserting an infinite conjunction of particular claims. To assert, for instance, that nothing travels faster than the speed of light in a vacuum is, looked at in this way, to assert that the speed of sound in air is not greater than the speed of light in a vacuum, and that the speed of that tennis ball is not greater than the speed of light in a vacuum, and so on. Similarly, Frege saw no difference in kind between particular and general logical claims - for him, they too differ only in their degree of generality. To assert that for all propositions  $p$  and  $q$ , if  $p$  and  $q$  then  $p$ , is to assert, for each pair of propositions  $p$  and  $q$ , that if  $p$  and  $q$  then  $p$ . In contrast, there seems to be a difference in kind between the general and particular claims of model-theoretic logic. The general claim that under all interpretations the schema  $p \& q \rightarrow p$  expresses a truth is a claim made from outside the logical system, not inside, whereas the specific claim that  $A \& B \rightarrow A$  (where  $A$  and  $B$  are sentence letters) is made inside the system, not outside. So to assert the general claim is not to assert anything inside the system, and so not to assert an infinite conjunction of more particular claims inside the system.

What does Frege mean by saying that logic is the science of the most general laws of truth? He means, I take it, that although every science is a science of the laws of truth, logic is the science of the *most general* laws of truth. Physics is the science of the laws of truth about speed, sound, light, vacuums, and so on; Chemistry is the science of the laws of truth about gases, electrons, inertness, and so on; but logic is the science of the laws of truth about anything whatsoever. He says that “we do not demand of [logic] that it should go into what is peculiar to each branch of knowledge and its subject matter. On the contrary, the task we assign logic is only that of saying what holds with the utmost generality for all thinking, whatever its subject matter.”<sup>5</sup> But why ‘laws of truth’? In what sense can he mean that every science is a science of the laws of truth? On the face of it that is false - the laws of physics are not laws of truth but laws of gravity, motion, and so on. But this is to think of laws as *descriptive* - as true descriptions of the way things are. There is another way to think of them, and this seems to be Frege’s way: as *prescriptive* - as what we *ought* to think if we are to think of the truth. He says, “Like ethics, logic can also be called a normative science. How must I think in order to reach the goal, truth”? And: “We could, with equal justice, think of the laws of geometry and the laws of physics as laws of thought or laws of judgement, namely as prescriptions to which our judgements must conform in a different domain if they are to remain in agreement with the truth.”<sup>6</sup>

As I have noted, physics and chemistry seek theories in which a set of laws can be used to explain particular truths. The fewer laws and the more that they can explain the better. Finding such theories is a matter of experiment (trial and error), and in cases where two theories are equally able to explain all particular truths, simplicity and elegance are used as evidence of which theory is the *true* one (assuming the natural view that scientific theories can be true or false). I think Frege took it that not only does logic aim to find laws (of truth), but, like physics and chemistry, aims to find the *true* logical theory - the set of logical laws that best explains the truth of all particular logical statements. How do we use logical laws to ‘explain’ logical truths? By deducing the logical truths from those laws, just as we ‘explain’ particular scientific truths by deducing them scientific laws. So the logical theory that Frege sees logic as aiming for is a deductive theory in which certain logical laws are taken as axioms from all logical truths can be deduced. Frege claims that in *The Foundations of Arithmetic* he “sought to make it probable that arithmetic is a branch of logic”<sup>7</sup>, that is, that all arithmetical truths can be derived from the laws of logic. I think we can understand Frege as making a claim about arithmetic similar to a claim that we might make about, say, ballistics: that the work of science makes it probable that ballistics is a branch of physics - that all ballistical truths can be derived from the laws of physics. We might think that science makes this probably true in this sense: that the most likely explanation for why so many ballistic truths have been shown to be derivable from the currently accepted laws of physics is that currently accepted physical theory is correct. I suggest that Frege thinks that a similar conclusion should be drawn from his work in *The Foundations of Arithmetic* - that the most likely explanation for why he is able to derive the arithmetical truths that he does in the way that he does is that in the correct logical theory all arithmetical truths can be derived from logical laws, that is, that arithmetic is a branch of logic.

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<sup>5</sup> L2.

<sup>6</sup> L2.

<sup>7</sup> GG, introduction to volume 1.

We know, and I take it that Frege knew, that there is more than one logical theory (deductive system) in which any given set of logical truths can be derived, and that this makes it difficult to maintain that there is one true logical theory. So on what evidence do I claim that this is what Frege believed? The following passage from the preface to *Grundgesetze* suggests that not just working but working well, or working better, are tests of his logical theory:

The whole of the second part is really a test of my logical convictions. It is prima-facie improbable that such a structure could be erected on a base that was uncertain or defective. Anyone who holds other convictions has only to create a similar structure upon them, and I think he will perceive that it does not work, *or at least does not work so well*. As a refutation of this I can only recognize someone's actually demonstrating either that a *better, more durable* edifice can be erected upon other fundamental convictions, or else that my principles lead to manifestly false conclusions.<sup>8</sup> (My emphasis)

He also suggests that what justifies taking a logical law to be an axiom and therefore not in need of proof is not that it is obviously true, or recognisable by us as true by reflecting upon what it means, but its serving as an axiom in the ultimate logical theory. Compare this with how we might justify taking quarks to be fundamental physical particles. The justification is not that we can recognise them as fundamental by reflecting upon their nature, but that our best physical theory says that they are fundamental. This is not to say that it is not because of what they are intrinsically like that they are fundamental, only that it is not knowing their intrinsic nature that justifies us as taking them to be fundamental, but the role they play in our best physical theory. So too, I think Frege thought that what *makes* a logical law an axiom is something intrinsic to it, but that what justifies us in taking it to be an axiom is that it plays the role of axiom in our best logical theory. Thus, just as which particles are fundamental is to be determined by physical experiment and the development of physical theory, which logical laws are axioms is to be determined by logical experiment and the development of logical theory. This is quite different to what seems the natural view - that what justifies taking a logical law as an axiom is that we *can* do so. Frege view is that what justifies it is that we *ought* to do so.

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<sup>8</sup> GG, xxv-xxvi.

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