Philosophy of Education: Essay

Wylie Breckenridge
October, 1995

Question 1

"S knows that p (where p is a proposition) if and only if p is true, S believes that p, and S is justified in believing that p." Discuss this claim.

Throughout this essay we will take S to stand for a person and p to stand for a proposition, that is, an utterance that is either true or false.

Some of the statements we make are of the form "S knows that p". We confidently decide when "S knows that p" and when "S doesn't know that p", so we obviously have an understanding of what knowledge is, but can we define it? We need to look at examples of statements of the form "S knows that p" and see if they have common features.

Firstly, we do not say "S knows that p" if p is false. If Peter says "the car is green" when in fact the car is red then we would not say "Peter knows that the car is green". We require p to be true before we claim "S knows that p".

There is another requirement. If I say "There is a pen in my top draw, but I'm not sure" then even if it was true we would not say that I know it, because we reserve the word 'know' only for those cases in which we are *certain*. Another way of saying "S is sure that p" is "S believes that p", where we are using 'believe' in the sense of "believes without doubt". This is important because we use the word 'believe' in many different ways. I may say "I believe that tomorrow will be 23^{o}C" or "I believe his name is Peter" but in these cases I actually mean "I'm pretty sure, but I could be wrong".

Despite being necessary these two requirements are not sufficient for knowledge, for there *are* cases in which S believes a true proposition but in which we would not grant "S knows that p". Suppose I say "I believe that today's name begins with T", and suppose that today is Thursday[1]. We would initially be prepared to grant that I *know* today's name begins with T. But if I was asked "how do you know" and I replied "Well, today is Tuesday and Tuesday begins with T, therefore today's name begins with T", then I could no longer claim to know it. It has become apparent that I do not have good reason to believe it, that my belief is unjustified, that it was no more than luck. For S to know that p, we require that S be able to *justify* belief in p.

Can we specify what is adequate justification? Can we set down rules for deciding whether S has justified belief that will cover all claims to knowledge? The justification that we give for our beliefs comes in many forms and space only allows for a cursorial discussion, but it will be sufficient to make a general point.

If I had said above that "today is Thursday and Thursday begins with T so today's name begins with T" then it would be granted that I know it. Here my justification is a valid

[1] Example adapted from that given in B.Russell, *The Problems of Philosophy*, Oxford University Press, London, 1976, p.77

deduction from true premises and this is generally regarded as adequate for knowledge. Well, almost, because not only must my premises be true but I must also *know* them: if I said “Today’s name begins with T and ‘Touch’ begins with T therefore today’s name begins with the same letter as ‘Touch’”, then I have made a valid deduction from true premises, but I could not claim to *know* the conclusion because it is only by luck that I started with the true premise “Today’s name begins with T”, since I did not *know* this. I say that this form of justification is *generally* regarded as adequate because there are philosophers, so called sceptics, who doubt the reliability of the deductive principles it relies upon.

Suppose I am in a room with several people who can see a pen in my hand. Suppose I claim “I believe there is a pen in my hand” and when asked to justify my belief I say “I can feel it and I can see it”. Then the others will grant that I *know* there is a pen in my hand. Here my justification is based on sense experience and in many cases this will be regarded as adequate for knowledge. But there is some doubt involved here because our senses can deceive us, and in fact it is not clear that we can *ever* trust them.

If I say “I know that the First Fleet arrived in Australia in 1788, because I read it in my History text book”, then I am justifying belief in this proposition by appealing to authority, in this case the authority of the text book. We would usually grant that this is sufficient justification because we trust that what the book says is true, probably because we assume that any mistakes would have been noticed by *someone* and been corrected. We also feel that, if we had the time and the need, we could check and confirm the truth of this claim for ourselves. We must make sure, however, that we appeal to a source that is indeed an authority on the particular topic and remember that doing so always involves an element of risk.

Suppose I claim that “I believe the sun will rise tomorrow” and when asked to justify this I explain “because it has risen every other day”. Here I am providing an inductive reason for my belief. Even if the Sun did in fact rise tomorrow we would not say that I *knew* it would. Inductive reasoning only provides us with a degree of confidence, never certainty, because we cannot be sure that the Universe and its objects will continue to behave as they have in the past. Although inductive reasoning often leads us to feel very confident about a proposition it is not sufficient to be able to claim knowledge.

There are some instances in which it seems unnecessary to ask S to justify belief in p. Consider the statement “I believe I feel hungry”. If asked why I believe this I can do little more than say “because I feel it”, which has added nothing more. Yet we *would* be prepared to grant that I know it.

There are other instances in which we might grant that “S knows that p” even though S can give no justification. For example, suppose a man can predict the fall of a dice without mistake. If, before the first roll of the dice, he claimed that “the next roll will be 2, because that number just popped into my head”, and if it was indeed 2, then we would dismiss it as mere coincidence. But if he continued to make correct predictions based on these numbers that pop into his head then we might eventually grant that he does indeed *know* what each roll will be, even though we do not know *how*, and even though he cannot justify it.

These last two examples do not fit well with our definition of knowledge in which S must provide justification. Some philosophers[2] replace the requirement that S be able to justify p with the requirement that S must earn *the right to be sure* about p. To earn this right, S may not have to justify p. The dice roller above has earned the right to be sure simply because he can make repeated accurate predictions.

If we do define "knowledge of p" to be "justified true belief in p" then these examples should show that the problem of deciding whether S knows that p becomes the problem of deciding whether S has justification for believing that p, and that no general rules can be set down for making this decision. What counts as adequate justification will depend upon the nature of p and each particular case must be examined by itself.

Question 2

> *How does the belief that education is concerned with propositional knowledge influence the way in which you teach your subject?*

Firstly, we need to discuss the knowledge of mathematical propositions. When can I claim to know that, for example, the angle sum of a triangle is 180^o?

I must specify whether I am making this claim in the realm of applied mathematics or pure mathematics. If I am claiming that if someone was to draw a triangle on a piece of paper, measure the angles and add them together then she would obtain 180^o, then I am making a claim about the physical world based upon a mathematical model and I am making this claim as an applied mathematician. Some models have been found to give results that are in close agreement with our observations of the world, but we cannot claim that they give us *knowledge* about the world because there is always the chance that a single disagreement will be found that will consequently show the model to be limited and the previous agreements to be no surer than mere coincidence. Such claims about the angle sum of physical triangles are no different to the claim that the sun will rise tomorrow; it is true for every triangle we have tried so far so we are confident, but we cannot be *sure* that it will be true for the next[3]. So we cannot claim to know any proposition about the world derived from a mathematical model.

On the other hand, if I am claiming that, given the axioms and definitions of Euclidean Geometry, the angle sum of triangle is 180^o, then I am making a claim within the logical system of geometry, without regard to the physical world. This is pure mathematics and we will see that here, unlike applied mathematics, knowledge is possible.

Geometrical propositions are true, within the system of Euclidean Geometry, if they can be derived by a chain of valid deductions (i.e. proven) from the axioms, definitions and previously proven propositions. So if I can provide such a proof for the angle sum of a triangle then (i) it is true, (ii) I will believe it (assuming I behave rationally) and (iii) I can justify it. If I also *know* the premises, that is the axioms, definitions and previous propositions, then I will *know* this angle sum proposition.

---

[2] A.J.Ayer, *The Problem of Knowledge*, Penguin, London, 1979, pp.31-35

[3] In fact, General Relativity, the model that we now use for the geometry of the Universe, predicts that *no* triangle in the vicinity of the Earth will have an angle sum of 180^o, and this has been supported by experiment.

Thus we *can* be confident of having knowledge of the propositions within an axiomatic system. Some care is needed though. Firstly, there have been many instances in mathematical history of inadequacies in proofs that have escaped the notice of even the finest mathematicians for many years. It would be unusual to question the truth of the simplest results, but wise to remain in some doubt about results that require complex proofs and for this doubt to be in proportion to the complexity of the proof. There are also the sceptics mentioned above who question the validity of the rules of inference and who have even go as far as to claim that we cannot know *any* mathematics.

So, a student cannot claim to know a mathematical proposition unless he can provide a valid proof from known premises. As we shall see there are several ways that he can fall short of this demand, and at least one of these will apply to most, if not all, students leaving our schools. It is a very rare student who can truthfully claim to know that the angle sum of a triangle is 180^{o}.

Firstly, the student may have been taught the result without having been given *any* justification, basing belief in the proposition on the *authority* of the teacher or textbook. As discussed above, relying on authority always leaves some doubt about the truth of a proposition.

Secondly, if the student *was* given justification it may not have been a deductive proof. One common way of justifying the proposition is to cut out a paper triangle, cut off the corners, place them adjacently and to observe that they form a straight line (180^{o}). Another is to get the students to draw several triangles and measure and add the angles. The students are then led to conclude the result by induction which, again, is not adequate justification for knowledge. Moreover, we cannot take observations from the physical world and deduce results within a mathematical system.

Thirdly, even if the student has seen a deductive proof he may not *know the premises*. The proof will rely upon previous propositions that, possibly for the above reasons, the student may not know. It will also draw upon axioms and definitions that the student will probably not have even seen, let alone know. In fact, I am a mathematics teacher and cannot remember having ever seen the axioms or definitions of Euclidean Geometry.

Can we teach our students so that they *know* the angle sum property of a triangle?

Time is one obstacle: it would take too long to begin geometry with its axiomatic origins, given the volume of mathematics that we must cover in school. Further, at the age at which students begin to learn geometry the concept of an axiom is far too abstract for them. Further still, it is difficult enough to stimulate their interest in geometry without weighing the subject down with logical rigour. What is most important at their stage in life is to be exposed to these results and to be able to use them. Putting the results on a more secure foundation can come later, when they are ready for more abstract work. The best we can do at school is to discuss the logical foundations whenever possible, to give the students some idea that the tower of mathematics is only as secure as its base.

Another problem is that the proofs of geometrical results can be difficult to demonstrate and understand, so that at the time a result is introduced we can do little more than provide some demonstration to bring the students to believe the result. For example, the process of cutting off the corners of a triangle and letting them form a straight line is in no way a proof of the angle sum property, but having seen it the students are quick to believe the result.

We should provide deductive proofs of propositions whenever we can. If we cannot, it is important to do two things: (i) to provide some kind of justification so that the students learn to expect it and to not accept results without questioning them, and (ii) to make it clear that because we have not proven the result it remains in some doubt.

Perhaps this is the most important part of our teaching: to encourage the students to question everything, to realise that every result they use is open to some doubt until they can prove it from known premises. As they develop mathematically they can come closer to this goal. In the meantime, however, most students *can* claim to know that *if* what the teacher or text book says is true *then* the angle sum of a triangle is 180^o, or that *if* the premises that the proof is based upon are true *then* the angle sum is 180^o. Over the years, teachers and text books have proven to be reliable sources of knowledge, so our students can feel confident when they claim that the angles of a triangle add up to 180^o.

Bibliography

A.J.Ayer, *The Problem of Knowledge*, Penguin, London, 1979.

J.Hospers, *An Introduction to Philosophical Analysis* 2nd Edition, Routledge and Kegan Paul, London, 1970.

B.Russell, *The Problems of Philosophy*, Oxford University Press, London, 1976.

W.G.S.Smith, "Education and Knowledge", Lecture Notes, Melbourne University, 1995.