

Lecture 8
QUANTIFIED MODAL LOGIC

Reading: Saul Kripke (1963), 'Semantical Considerations on Modal Logic', *Acta Philosophica Fennica* **16**, pp. 83-94.

1. So far we have been considering sentences involving 'It is necessary that' ('necessarily') and 'It is possible that' ('possibly'), but not involving quantifier expressions such as 'everything' and 'something'. That is, we have been considering *non-quantified* modal logic. We now consider *quantified* modal logic. We shall consider sentences like the following:

- a. Possibly something is red.
- b. Something is possibly red.

2. According to the approach we have been taking, 'Possibly something is red' means that there is a possible world in which something is red, and 'Something is possibly red' means that something is such that there is a possible world in which it is red. If we add the usual quantifiers to S5 to get the language S5Q, they can be translated as follows:

- a. $\Diamond \exists x R x$
- b. $\exists x \Diamond R x$

3. This raises an important question: Should there be just *one* domain of things over which the quantifiers range (in every possible world)? Or should there be a domain for each possible world, with possibly different domains from world to world?

4. On the first approach, call it the *fixed domain* approach, we can say that 'Possibly something is red' means that there is a possible world in which something in the domain is red. On the second approach, call it the *variable domain* approach, we need to say that 'Possibly something is red' means that there is a possible world in which something in the domain of that world is red.

5. How do we decide between the two approaches? Well, the choice makes a difference. For example, it effects whether or not the following two sequents are correct (the Barcan sequent and its converse):

- a. $\Diamond \exists x F x \vdash \exists x \Diamond F x$ (Equivalently: $\forall x \Box F x \vdash \Box \forall x F x$)
- b. $\exists x \Diamond F x \vdash \Diamond \exists x F x$ (Equivalently: $\Box \forall x F x \vdash \forall x \Box F x$)

On the fixed domain approach, (a) and (b) are correct; on the variable domain approach, (a) and (b) are incorrect.

This means it effects whether or not the following two argument forms count as valid:

- c. Possibly something is F, therefore something is possibly F. (Equivalently: Everything is necessarily F, therefore necessarily everything is F.)
- d. Something is possibly F, therefore possibly something is F. (Equivalently: Necessarily everything is F, therefore everything is necessarily F.)

6. Do we want these sequents to come out as correct, and these argument forms to come out as valid? It seems not. The following instance of (5c) seems to be an invalid argument:

- a. Possibly someone was Wittgenstein's child, therefore someone was possibly Wittgenstein's child.
- b. $\Diamond \exists x R x \vdash \exists x \Diamond R x$ (where 'Rx' translates 'x is a child of Wittgenstein'.)

Thus it seems that we want the Barcan sequent to be incorrect, so that the variable domain approach is the right approach.

7. Furthermore, the following instance of (5d) seems to be an invalid argument:

- a. Necessarily everything exists, therefore everything necessarily exists.
- b. $\Box \forall x Ex \not\models \forall x \Box Ex$ (where 'Ex' translates 'x exists'.)

Thus it seems that we want the converse Barcan sequent to be incorrect, so that the variable domain approach is the right approach.

8. Here are two more reasons to think that the variable domain approach is the right approach:

- a. Intuitively, it is true that there could have been more things than there are. The variable domain approach allows this, but the fixed domain approach does not.

How so? Paraphrase the claim as, 'Possibly there is something which is not one of these', where 'these' rigidly designates all the actual things. Then we can translate this into S5Q as ' $\Diamond \exists x Rx$ ', where ' Rx ' translates ' x is not one of these'. On the variable domain approach there are interpretations in which this comes out true, but on the fixed domain approach there are not.

- b. Intuitively, it is true that there could have been *fewer* things than there are. The variable domain approach allows this, but the fixed domain approach does not.

How so? Paraphrase the claim as, 'There is something such that possibly nothing is it'. Then we can translate this into S5Q as ' $\exists x \Diamond \neg \exists y (x = y)$ '. On the variable domain approach there are interpretations in which this comes out true, but on the fixed domain approach there are not.

9. Kripke (in the reading) assumes that the variable domain approach is the right approach, and accordingly gives a semantics for S5Q.

10. Williamson (in the reading for next time) argues that, contrary to what seems to be the case, the fixed domain approach is the right approach.