PHIL 331/MATH 281: Selected Answers for Week 10

Five key results

b ‘¬∃υφ’ is true on an interpretation I iff ‘∀υ¬φ’ is true on I.

Proof. Suppose that ‘¬∃υφ’ is true on an interpretation I. So for every assignment α, ‘¬∃υφ’ is true on I relative to α. So for every assignment α, ‘∃υφ’ is false on I relative to α. So for every assignment α, there is no υ-variant α' of α such that φ is true on I relative to α'. So for every assignment α, and every υ-variant α' of α, φ is false on I relative to α'. So for every assignment α, and every υ-variant α' of α, ‘¬φ’ is true on I relative to α'. So for every assignment α, ‘∀υ¬φ’ is true on I relative to α. So ‘∀υ¬φ’ is true on I. Now suppose that ‘∀υ¬φ’ is true on an interpretation I. So for every assignment α, ‘∀υ¬φ’ is true on I relative to α. So for every assignment α, and every υ-variant α' of α, ‘¬φ’ is true on I relative to α'. So for every assignment α, and every υ-variant α' of α, φ is false on I relative to α'. So for every assignment α, there is no υ-variant α' of α such that φ is true on I relative to α'. So for every assignment α, ‘∃υφ’ is false on I relative to α. So for every assignment α, ‘¬∃υφ’ is true on I relative to α. So ‘¬∃υφ’ is true on I.■

Models and consistency

a.iii {‘∀*x*∃*y*(P*y* ∧ ¬*x* = *y*)’, ‘∃*x*∃*y*((P*x* ∧ P*y*) ∧ ¬*x* = *y*)’} is consistent.

Proof. Let I be the following interpretation. The domain is {1, 2}; ‘P’ denotes {1, 2}. Then for everything in the domain there is some distinct thing in the domain which is in the set denoted by ‘P’, so ‘∀*x*∃*y*(P*y* ∧ ¬*x* = *y*)’ is true on I. And there are (at least) two (distinct) things in the domain which are in the set denoted by ‘P’, so ‘∃*x*∃*y*((P*x* ∧ P*y*) ∧ ¬*x* = *y*)’ is true on I. So there is an interpretation on which every wff in {‘∀*x*∃*y*(P*y* ∧ ¬*x* = *y*)’, ‘∃*x*∃*y*((P*x* ∧ P*y*) ∧ ¬*x* = *y*)’} is true, and the set is consistent. ■

b.iii {‘∃*x*R*xx*’, ‘¬a = b’, ‘∀*x*∀*y*(R*xy* → ((*x* = a ∧ *y* = b) ∨ (*x* = b ∧ *y* = a)))’} is inconsistent.

Proof. Suppose that there is an interpretation I on which every wff in this set is true. Since ‘∃*x*R*xx*’ is true on I, there is something in the domain of I, call it Bob, such that <Bob, Bob> is in the set denoted by ‘R’. Since ‘∀*x*∀*y*(R*xy* → ((*x* = a ∧ *y* = b) ∨ (*x* = b ∧ *y* = a)))’ is true on I, ‘(R*xy* → ((*x* = a ∧ *y* = b) ∨ (*x* = b ∧ *y* = a)))’ is true on I no matter what is assigned to ‘*x*’ and ‘*y*’. In particular, it is true on I when Bob is assigned to both ‘*x*’ and ‘*y*’. Since ‘R*xy*’ is true on I when Bob is assigned to ‘*x*’ and ‘*y*’, we must have that ‘((*x* = a ∧ *y* = b) ∨ (*x* = b ∧ *y* = a))’ is true on I when Bob is assigned to ‘*x*’ and ‘*y*’. So when Bob is assigned to ‘*x*’ and ‘*y*’, either ‘*x* = a’ and ‘*y* = b’ are both true on I, or ‘*x* = b’ and ‘*y* = a’ are both true on I. But ‘*x* = a’ and ‘*y* = b’ are both true on I when Bob is assigned to ‘*x*’ and ‘*y*’ only if ‘a’ and ‘b’ both denote Bob on I, and similarly ‘*x* = b’ and ‘*y* = a’ are both true on I when Bob is assigned to ‘*x*’ and ‘*y*’ only if ‘a’ and ‘b’ both denote Bob on I. So we must have that ‘a’ and ‘b’ both denote Bob on I. But then ‘¬a = b’ is false on I, which contradicts the assumption that it is true on I. So there is no such interpretation, and the set is inconsistent. ■

Extending the tableau system to QC

g. Done on the board
h. Done on the board