

PHIL 331/MATH 281: Selected Answers for Week 8

The syntax of QC

- a.i 'a' is **not** a wff
- a.iii 'F<sup>2</sup>ax' is a wff
- a.v 'Fx = Gy' is **not** a wff
- a.vii 'A<sup>2</sup>PQ' is **not** a wff
- a.ix ' $\forall x(Fax \rightarrow \exists xFax)$ ' is a wff
- a.xi ' $\forall a(\forall yFyy \vdash Faa)$ ' is **not** a wff
- a.xiii ' $\forall x(Fab \wedge (Ga \rightarrow \exists x(Fxb \wedge Gx)))$ ' is a wff
- a.xv ' $\exists x \wedge \exists y Fxy$ ' is **not** a wff

Unique decomposition

- a.iv In ' $\neg \forall x(Ax \rightarrow B)$ ' the main connective is ' $\neg$ ' and its immediate constituent is ' $\forall x(\mathbf{Ax} \rightarrow \mathbf{B})$ '.
- a.v In ' $(A \rightarrow \exists z(Pz \rightarrow \forall wPw))$ ' the main connective is ' $\rightarrow$ ' and its immediate constituents are '**A**' and ' $\exists z(\mathbf{Pz} \rightarrow \forall w\mathbf{Pw})$ '.

Scope and bondage

- a.iii ' $(\forall y \exists x Axy \vee \neg \forall x Ayx)$ ': all occurrences of variables are bound, except for the last occurrence of 'y'.
- b.iii ' $(Px \vee \exists x(Qx \wedge Rx))$ ': 'x' is a free variable, because it has an unbound occurrence (the first occurrence).
- c.i ' $\forall xAx$ ' is **closed**
- c.ii ' $\exists yFyy$ ' is **closed**
- c.iii ' $\exists x \neg Ba$ ' is **closed**
- c.iv ' $\forall x(Bx \vee Cxy)$ ' is **open** ('y' has an unbound occurrence)
- c.v ' $\exists y(\exists x \neg Ba \leftrightarrow \forall x(Bx \vee Cxy))$ ' is **closed**
- c.vi ' $(\forall x(Bx \vee Cxy) \wedge \exists y(\exists x \neg Ba \leftrightarrow \forall x(Bx \vee Cxy)))$ ' is **open** ('y' has an unbound occurrence)
- c.vii ' $(A \rightarrow \exists z(Pz \rightarrow \forall wPw))$ ' is **closed**

Substitution

- a.v ' $(\forall xAxcy \wedge Bxz)$ '('a'/'x') = ' $(\forall xAxcy \wedge Baz)$ '; 'x' is free for 'a'.