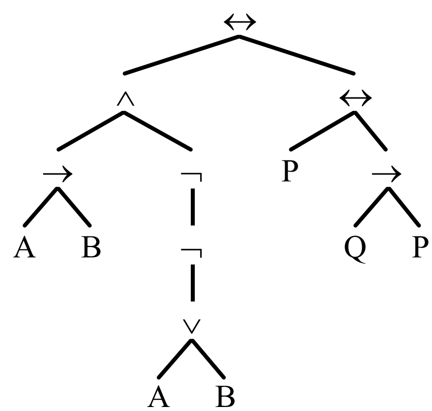
PHIL 331/MATH 281: Answers to First Test

1. a. **No** (missing brackets)
   b. **Yes**
   c. **Yes**
   d. **No** (not standard notation)
   e. **Yes**
   f. **No** (‘⊨’ is not a symbol of PC)
   g. **No** (there are too many right brackets)
   h. **No** (should not have brackets around ‘Q’)
   i. **No** (‘φ’ and ‘ψ’ are not symbols of PC)
   j. **Yes**

2.



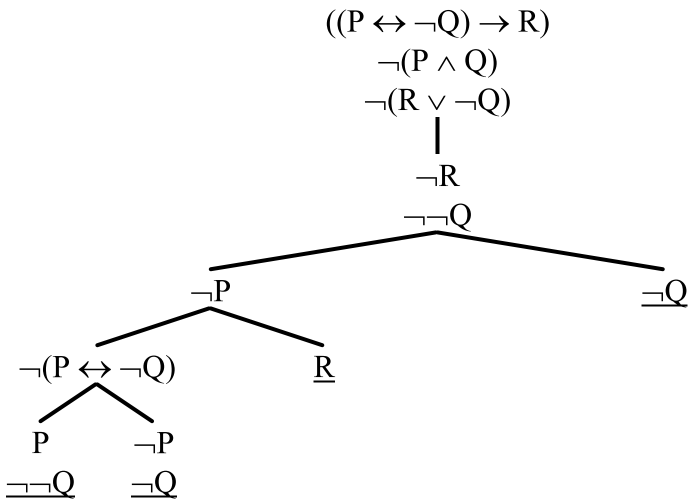
3. The wff is neither, because in its final column it has some ‘T’ and some ‘F’:

(¬	(A	∧	B)	→	(A	∨	(A	↔	B)))
F	T	T	T	**T**	T	T	T	T	T
T	T	F	F	**T**	T	T	T	F	F
T	F	F	T	**F**	F	F	F	F	T
T	F	F	F	**T**	F	T	F	T	F

4. a. **True** If φ is a tautology then on every interpretation φ is true, so on every interpretation ‘¬φ’ is false, so there is no interpretation on which ‘¬φ’ is true, so ‘¬φ’ is inconsistent.
   b. **False** {‘(A ∨ ¬A)’, ‘B’, ‘¬B’} contains a tautology but is inconsistent.
   c. **False** The tableau which consists of just the root wff ‘(A ∧ ¬A)’ has an open branch but its root wffs are inconsistent (the open branch is not *fully developed*).
   d. **True** By definition of soundness.
   e. **False** ‘consistent’ should read ‘inconsistent’.

5. Suppose that ‘$((P \vee Q) \rightarrow R)$’ is false. Then ‘$(P \vee Q)$’ is true and ‘R’ is false. Since ‘$(P \vee Q)$’ is true, either ‘P’ is true or ‘Q’ is true (or both). If ‘P’ is true, then since ‘R’ is false we have that ‘$(P \rightarrow R)$’ is false. If ‘Q’ is true, then since ‘R’ is false we have that ‘$(Q \rightarrow R)$’ is false. Either way, ‘$((P \rightarrow R) \wedge (Q \rightarrow R))$’ is false. Now suppose that ‘$((P \rightarrow R) \wedge (Q \rightarrow R))$’ is false. Then either ‘$(P \rightarrow R)$’ is false or ‘$(Q \rightarrow R)$’ is false (or both). If ‘$(P \rightarrow R)$’ is false then ‘P’ is true and ‘R’ is false. If ‘$(Q \rightarrow R)$’ is false then ‘Q’ is true and ‘R’ is false. Either way, ‘$(P \vee Q)$’ is true and ‘R’ is false, so ‘$((P \vee Q) \rightarrow R)$’ is false. ■

6. Tableau closes so the sequent is correct:



7. Suppose that φ, ψ and χ are wffs such that $\varphi \models \psi$ and $\psi \models \chi$. Suppose that I is an interpretation on which φ is true. Since $\varphi \models \psi$, this is an interpretation on which ψ is true. Since $\psi \models \chi$, it is an interpretation on which χ is true. So any interpretation on which φ is true is one on which χ is true. So $\varphi \models \chi$. ■

8. We will prove by induction that for every $n \geq 0$, every wff with n connectives has the same number of left and right brackets. **Base case**. Suppose that φ is a wff with 0 connectives. Then φ is a sentence letter. So φ has the same number of left and right brackets (zero). So the result is true for $n = 0$. **Inductive step**. Suppose the result is true for $n = 0, \ldots, k$, for some $k \geq 0$, and suppose that φ is a wff with $k+1$ connectives. Since $k + 1 > 0$, φ has at least one connective, so it is a complex wff. So either φ = ‘$\neg\psi$’, for some wff ψ which has k connectives, and hence for which the result is true, or φ = ‘$(\psi \ C \ \chi)$’, for some binary connective C, and some wffs ψ and χ, each of which has no more than k connectives, and hence for which the result is true. Suppose that φ = ‘$\neg\psi$’. Since the result is true for ψ, ψ has the same number of left and right brackets, and hence so does φ (it has the same number of each). Suppose that φ = ‘$(\psi \ C \ \chi)$. Since the result is true for ψ and χ, they contain the same total number of left and right brackets, and hence so does φ (it has one more of each). Either way, φ has the same number of left and right brackets. So the result is true for $n = k + 1$. ■