PHIL 331/MATH 281: Answers to Second Test

1. a. '$\leftrightarrow$'
   b. '$\forall x(\mathrm{G}xy \rightarrow x = y)$'
   c. Free (it has at least one free occurrence)
   d. No (one free occurrence of 'x' becomes a bound occurrence of 'y')
   e. '$(\exists x\mathrm{F}xy \wedge \exists y\mathrm{G}a y)$'

2. a. True
   b. True
   c. True
   d. True
   e. True

3. a. True
   b. False
   c. False
   d. False
   e. False

4. Suppose that '$\exists x\varphi$' is true on an interpretation I.
   So for every α, '$\exists x\varphi$' is true on I relative to α.
   So for every α, there is some α' such that φ is true on I relative to α'.
   So for every α, there is some α' such that '$\neg\varphi$' is false on I relative to α'.
   So for every α, it is not the case that for every α', '$\neg\varphi$' is true on I relative to α'.
   So for every α, it is not the case that '$\forall x\neg\varphi$' is true on I relative to α.
   So for every α, '$\forall x\neg\varphi$' is false on I relative to α.
   So for every α, '$\neg\forall x\neg\varphi$' is true on I relative to α.
   So '$\neg\forall x\neg\varphi$' is true on I. ■

5. The following tableau has a fully developed open branch, so the root wffs are consistent:

   '$\neg$Fa'
   ~~'$\exists x(\mathrm{F}x \wedge \forall y\mathrm{G}y)$'~~
   |
   ~~'$(\mathrm{F}b \wedge \forall y\mathrm{G}y)$'~~
   |
   'Fb'
   '$\forall y\mathrm{G}y$'
   |
   Ga
   Gb

   Here is a model: the domain is {1, 2}; 'a' denotes 1; 'b' denotes 2; 'F' denotes {2}; 'G' denotes {1, 2}.

6. Let I be the following interpretation of QC: the domain is {1, 2}; 'F' denotes {<1, 1>, <2, 2>}. Then '$\forall x \exists y F x y$' is true on I (because for each thing in the domain there is an 'F' arrow from it to something), but '$\exists x \forall y F x y$' is false on I (because it is not the case that there is something in the domain with an 'F' arrow from everything to it). So there is an interpretation on which these wffs have different truth values, so they are not logically equivalent.

7. The following tableau closes, so the root wffs are inconsistent, and the sequent is correct: