PHIL 331/MATH 281: First Test

Time allowed: 75mins
Do every question. Each question is worth 5 marks.

1. For each of the following, say whether or not it is a wff of PC (no need to prove it):
   a. ‘$(A \vee B \wedge C)$’
   b. ‘$(P \rightarrow (Q \rightarrow R))$’
   c. ‘$\neg\neg\neg\neg\neg\neg A$’
   d. ‘$\leftrightarrow A \neg B$’
   e. ‘$\neg(A \wedge B)$’
   f. ‘$(P \wedge Q) \models P$’
   g. ‘$(((S \vee T) \rightarrow S) \leftrightarrow (T \wedge S)))$’
   h. ‘$(P \rightarrow \neg(Q))$’
   i. ‘$(\varphi \wedge \psi)$’
   j. ‘X’

2. Draw a tree to show the constituent structure of the following wff:
   ‘$(((A \rightarrow B) \wedge \neg\neg(A \vee B)) \leftrightarrow (P \leftrightarrow (Q \rightarrow P)))$’

3. Construct a truth table for the following wff and use it to decide whether the wff is a tautology, a contradiction, or neither:
   ‘$(\neg(A \wedge B) \rightarrow (A \vee (A \leftrightarrow B)))$’

4. For each of the following say whether it is true or false (no need to prove it):
   a. For every wff φ: if φ is a tautology then ‘$\neg\varphi$’ is inconsistent
   b. Any set of wffs that contains a tautology is consistent
   c. Any tableau with an open branch has consistent root wffs
   d. The tableau system is sound just in case for all Γ and φ: if $\Gamma \vdash \varphi$ then $\Gamma \models \varphi$
   e. The compactness theorem says that for all Γ: if Γ is consistent then Γ has some finite subset that is consistent

5. Prove that the following two wffs are logically equivalent by arguing that there is no interpretation on which they take different truth values:
   ‘$((P \vee Q) \rightarrow R)$’ and ‘$((P \rightarrow R) \wedge (Q \rightarrow R))$’

– Start a new answer booklet –

6. Use the tableau method to decide whether or not the following is a correct semantic sequent:
   ‘$((P \leftrightarrow \neg Q) \rightarrow R)$’, ‘$\neg(P \wedge Q)$’ $\models$ ‘$(R \vee \neg Q)$’

7. Prove that if φ, ψ and χ are wffs such that $\varphi \models \psi$ and $\psi \models \chi$, then $\varphi \models \chi$ (do not use induction).

8. Prove by induction that every wff of PC has the same number of left and right brackets.

– End of test –