PHIL 331/MATH 281: Second Test

Time allowed: 75mins
Do every question. Each question is worth 5 marks.

1. Answer the following (no need to give reasons):
   a. What is the main connective in ‘$(\neg\exists x\mathrm{F}x \leftrightarrow (\mathrm{Fa} \rightarrow (\mathrm{P} \wedge \forall y\mathrm{G}y)))$’?
   b. What is the scope of the occurrence of ‘$\forall x$’ in ‘$(\exists x\mathrm{F}x \rightarrow \exists y\forall x(\mathrm{G}xy \rightarrow x = y))$’
   c. Is ‘z’ free or bound in ‘$(\forall z\mathrm{F}z \vee (\exists z(\mathrm{G}z \wedge \mathrm{H}z) \rightarrow \mathrm{G}z))$’?
   d. Is ‘x’ free for ‘y’ in ‘$(\forall y\mathrm{G}xy \rightarrow \exists z\mathrm{G}xz)$’?
   e. What is ‘$(\exists x\mathrm{F}xy \wedge \exists y\mathrm{G}xy)$’(‘a’/‘$x$’)?

2. Let I be the following interpretation of QC: the domain is the set of natural numbers, ‘a’ denotes 2, ‘E’ denotes the set of even numbers, ‘L’ denotes the set $\{\langle x, y\rangle: x < y\}$. Let α be an assignment such that α(‘x’) = 3. Find the truth values of the following wffs on I relative to α (no need to give reasons):
   a. ‘$\neg \mathrm{a} = x$’
   b. ‘$(\mathrm{Ea} \wedge \mathrm{La}x)$’
   c. ‘$\exists x(\mathrm{La}x \wedge \mathrm{E}x)$’
   d. ‘$(\mathrm{E}x \rightarrow \neg\mathrm{E}x)$’
   e. ‘$\forall x\neg\exists x\forall x\mathrm{E}x$’

3. For each of the following say whether it is true or false (no need to prove it):
   a. If a closed wff is true on an interpretation relative to one sequence then it is true on that interpretation relative to every sequence.
   b. For every open wff φ and interpretation I: φ is neither true nor false on I.
   c. We require the domain of an interpretation to be non-empty because the set denoted by each predicate letter must be non-empty.
   d. The tableau system gives us an effective procedure for determining whether or not a set of wffs is consistent.
   e. Because the domain of an interpretation can be infinitely large, the compactness theorem no longer holds for QC.

4. Prove from first principles that if ‘$\exists x\varphi$’ is true on an interpretation I then ‘$\neg\forall x\neg\varphi$’ is true on I.

– Start a new answer booklet –

5. Prove that {‘$\neg\mathrm{Fa}$’, ‘$\exists x(\mathrm{F}x \wedge \forall y\mathrm{G}y)$’} is a consistent set of wffs by constructing an appropriate tableau. Use the tableau to specify a model for the set.

6. Prove that ‘$\forall x\exists y\mathrm{F}xy$’ and ‘$\exists x\forall y\mathrm{F}xy$’ are not logically equivalent by specifying an interpretation on which one is true and the other is false (and explain why).

7. Use the tableau method to prove that the following is a correct semantic sequent:
   ‘$\exists x\forall y(\mathrm{F}y \leftrightarrow y = x)$’, ‘$(\mathrm{Fa} \wedge \mathrm{Fb})$’ $\models$ ‘a = b’

– End of test –