

PHIL 331/MATH 281: Week 12

Formalizing English sentences

1. Sometimes we give an interpretation of QC by specifying a domain and then pairing the individual constants and predicate letters with English language expressions:

Domain = everything

'i': 'Ithaca'

'C': 'is a U.S. city'

'G': 'is gorgeous'

'W': 'is west of'

'B': 'is between'

2. The idea is this:

'Ithaca' determines a thing in the domain (Ithaca), 'is a U.S. city' determines a set of things in the domain (the set $\{x: x \text{ is a U.S. city}\}$), 'is gorgeous' determines a set of things in the domain (the set $\{x: x \text{ is gorgeous}\}$), 'is west of' determines a set of pairs of things in the domain (the set $\{<x, y>: x \text{ is west of } y\}$), and 'is between' determines a set of triples of things in the domain (the set $\{<x, y, z>: x \text{ is between } y \text{ and } z\}$).

We are interpreting 'i' as denoting whatever thing in the domain 'Ithaca' determines, 'C' as denoting whatever set of things in the domain 'is a U.S. city' determines, 'G' as denoting whatever set of things in the domain 'is gorgeous' determines, 'W' as denoting whatever set of pairs of things in the domain 'is west of' determines, and 'B' as denoting whatever set of triples of things in the domain 'is between' determines.

3. Given this interpretation we can formalize 'Ithaca is gorgeous' as 'Gi'. That is, 'Ithaca is gorgeous' is true iff 'Gi' is true.

Proof. 'Gi' is true on this interpretation iff the thing denoted by 'i' is a member of the set denoted by 'G'; that is, iff the thing determined by 'Ithaca' is a member of the set determined by 'is gorgeous'; that is, iff $Ithaca \in \{x: x \text{ is gorgeous}\}$; that is, iff Ithaca is gorgeous; that is, iff 'Ithaca is gorgeous' is true. ■

4. Here are some other formalizations (on this interpretation):

- 'Everything is gorgeous' $\Rightarrow \forall xGx$
- 'Something is gorgeous' $\Rightarrow \exists xGx$
- 'Every U.S. city is gorgeous' $\Rightarrow \forall x(Cx \rightarrow Gx)$
- 'Some U.S. city is gorgeous' (or 'There is a gorgeous U.S. city') $\Rightarrow \exists x(Cx \wedge Gx)$
- 'Every U.S. city is west of Ithaca' $\Rightarrow \forall x(Cx \rightarrow Wxi)$
- 'No U.S. city west of Ithaca is gorgeous' $\Rightarrow \neg \exists x((Cx \wedge Wxi) \wedge Gx)$ or $\forall x((Cx \wedge Wxi) \rightarrow \neg Gx)$
- 'Ithaca is the only gorgeous U.S. city' $\Rightarrow (Ci \wedge Gi) \wedge \forall x((Cx \wedge Gx) \rightarrow x = i)$

- ‘There are at least two gorgeous things’ $\Rightarrow \exists x \exists y ((Gx \wedge Gy) \wedge \neg x = y)$
- ‘There is a U.S. city which is west of every other U.S. city’ $\Rightarrow \exists x (Cx \wedge \forall y ((Cy \wedge \neg x = y) \rightarrow Wxy))$

5. Often the choice of variables in the formalizations is not important.
6. If we took the domain of this interpretation to be the set of U.S. cities then we could simplify some of the formalizations. We could, for example, formalize ‘Every U.S. city is gorgeous’ as ‘ $\forall x Gx$ ’ (rather than as ‘ $\forall x (Cx \rightarrow Gx)$ ’).

7. Exercises

Consider the following interpretation of QC: the domain is the set of natural numbers, ‘E’: ‘is even’, ‘I’: ‘is interesting’, ‘L’: ‘is less than’, and ‘B’: ‘is between’.

- a. Formalize:
 - i. Every number is less than or equal to itself
 - ii. Some number is less than every other number
 - iii. If any number is even then every number is even
 - iv. If every number is even then any number is even
 - v. No interesting number is between two even numbers, unless it is itself even
 - vi. Any number with the property that all smaller numbers are interesting is itself interesting.
 - vii. There are at least two interesting numbers
 - viii. There are at most two interesting numbers
 - ix. Between every two interesting numbers there is an interesting number
 - x. For every number there is a unique smallest number which is greater than it.
- b. Translate:
 - i. $\exists x (\neg Ex \wedge \neg Lx)$
 - ii. $\forall x \forall y \forall z ((Bxyz \wedge Bzyx) \rightarrow x = z)$
 - iii. $\exists x (Ix \wedge \forall y (Iy \rightarrow y = x) \wedge Ex)$
 - iv. $\forall x \forall y (\forall w \forall z (Bzxy \leftrightarrow Bwxy) \rightarrow x = y)$
 - v. $\forall x ((Ex \wedge Lx) \rightarrow \neg \forall y ((Ey \wedge Iy) \rightarrow \neg Lxy))$

Formalizing English arguments

1. Just as for PC, QC gives us a way to prove that an English argument is valid: formalize the argument as a sequent about QC; if the sequent is correct then the argument is valid.

But the converse still does not hold: even if the sequent is not correct the argument might still be valid (it's validity might be due to features that we cannot capture in QC – QC has limited expressive resources).

2. Exercises

Prove that the following arguments are valid by formalizing them as sequents about QC and proving that the sequents are correct:

- a. Albert kissed Sue, therefore someone kissed Sue.
- b. Socrates is a man. All men are mortal. So Socrates is mortal.
- c. Any friend of Martin is a friend of John; Peter is not John's friend; hence, Peter is not Martin's friend.
- d. All human beings are rational; some animals are human beings; hence, some animals are rational.
- e. The happiest student loves the richest student. Anyone who loves another is fortunate. So, if the happiest student is not fortunate then nor is the richest student.
- f. Ed has just one sister and just one brother. He is his brother's favourite child. So Ed's father's favourite child has exactly two siblings.
- g. All the male chimpanzees can solve every problem. There's at least one problem. Any chimpanzee who can solve a problem will get a banana. Sultan is a male chimpanzee. Therefore, Sultan will get a banana.
- h. Sultan and Chica can solve exactly the same problems. If Sultan can solve any of the problems then he will get a banana. Sultan will not get a banana. Therefore Chica cannot solve any of the problems.
- i. Not all chimpanzees are trying equally hard. No chimpanzee is trying harder than himself. Therefore there are at least two chimpanzees.
- j. Among all the chimpanzees only Sultan is male. The chimpanzees who will get a banana are the males. Therefore Sultan is the chimpanzee who will get a banana.