PHIL 331/MATH 281: Week 7

Formalizing English sentences

1. Sometimes we interpret a sentence letter not by explicitly assigning it a truth value T or F, but by pairing it with an English sentence:

   'A': 'Grass is green'

2. What we are doing is interpreting 'A' as having the same truth value as 'Grass is green', whatever that may be. This is an alternative way of specifying a truth value of 'A' – a different **mode of presentation** of a truth value. Say that we **interpret** 'A' as 'Grass is green'.

3. We are not specifying that 'A' *abbreviates* 'Grass is green', and we are not specifying that 'A' has the same *meaning* as 'Grass is green'.

4. If we interpret 'A' as 'Grass is green', then 'Grass is green' is true iff 'A' is true. Say that we can **formalize** 'Grass is green' as 'A'.

5. In general, if S is an English language sentence and φ is wff of PC, say that we can **formalize** S as φ just in case S is true iff φ is true. If we can formalize S as φ, say that φ is **a logical form** of S.

   So 'A' is a logical form of 'Grass is green' (as long as we interpret 'A' as 'Grass is green').

6. Suppose we interpret 'A' as 'Grass is green' and 'B' as 'Snow is white'. Then we can formalize 'Grass is green and snow is white' as '(A ∧ B)'.

   Proof. '(A ∧ B)' is true iff 'A' is true and 'B' is true; that is, iff 'Grass is green' is true and 'Snow is white' is true; that is, iff grass is green and snow is white; that is, iff 'Grass is green and snow is white' is true. ■

   So if 'A' and 'B' are interpreted in this way, then '(A ∧ B)' is a logical form of 'Grass is green and snow is white'.

7. The indefinite article is important here – this is *a* logical form, not *the* logical form. 'Grass is green and snow is white' has other logical forms. It also has the logical form 'C', if we interpret 'C' as 'Grass is green and snow is white'.

8. Suppose that S and T are English language sentences, that φ and ψ are wffs of PC, and that we interpret φ as S and ψ as T. Then we can formalize:

   a. 'It is not the case that S' as '¬φ'
   b. 'S and T' as '(φ ∧ ψ)'

c. ‘S or T’ as ‘$(\varphi \vee \psi)$’
d. ‘If S then T’ as ‘$(\varphi \rightarrow \psi)$’
e. ‘S just in case T’ as ‘$(\varphi \leftrightarrow \psi)$’
f. ‘S if T’ as ‘$(\psi \rightarrow \varphi)$’
g. ‘S only if T’ as ‘$(\varphi \rightarrow \psi)$’
h. ‘S unless T’ as ‘$(\varphi \vee \psi)$’
i. ‘S but T’ as ‘$(\varphi \wedge \psi)$’

(Each of these needs proof.)

9. Formalizing ‘If S then T’ as ‘$(\varphi \rightarrow \psi)$’ is controversial. The claim is that the truth value of ‘If S then T’ is determined in the following way:

S	T	‘If S then T’
T	T	T
T	F	F
F	T	T
F	F	T

The second row is not controversial. But the other three are.

Here is some justification. We want ‘If S and T then S’ to come out true no matter what the truth values of S and T. This gives us the other three rows.

Here is some more. Consider the claim, ‘For every *x*, if *x* is F then *x* is G’. We want cases in which *x* is F and *x* is G to be confirming examples of this claim – this gives us row one. And we don’t want cases in which *x* is not F to be counterexamples to this claim – this gives is rows three and four.

The main thing is that formalizing ‘Is S then T’ as ‘$(\varphi \rightarrow \psi)$’ is **safe** – it will not lead us to declaring as valid any argument that is not actually valid (this needs proof).

10. Because the wffs of PC are structurally unambiguous, any English sentence that is structurally ambiguous must be disambiguated before it can be formalized. So formalizing can force disambiguation.

Example: ‘Albert kissed Sue and Bert thanked Sue or Chris spoke to Sue’ can be understood in two different ways. If we interpret ‘A’ as ‘Albert kissed Sue’, ‘B’ as ‘Bert thanked Sue’ and ‘C’ as ‘Chris spoke to Sue’, then the two readings can be formalized as the following wffs, which are not logically equivalent:

‘$(A \wedge (B \vee C))$’
‘$((A \wedge B) \vee C)$’

11. Exercises

Formalize the following sentences as wffs of PC as informatively as possible (remember to specify an interpretation of the sentence letters used):

a. If it's raining it must be cloudy.
b. There are at least three donkeys.
c. I won't go unless you come with me.
d. Alice went to the party, but Barbara didn't.
e. If everyone is happy then someone is happy.
f. If someone enjoyed that play I'll be surprised.
g. A sufficient condition for *x* to be odd is that *x* is prime.
h. If anyone enjoyed that play he or she is easy to please.
i. Fiorello goes to the movies only if a comedy is playing.
j. Alice will both go for a run or visit her parents and study.
k. Smokey is an aggressive Grizzly Bear with an injured ear.
l. Alice will either go for a run and study or go visit her parents.
m. Alice will go for a run and either study or go visit her parents.
n. Alice, who was my oldest friend in Boston, left the party early.
o. If John and Mary go to the movies then John will go to the movies.
p. Karpov will win the chess tournament unless Kasparov wins today.
q. A necessary condition for a sequence *s* to converge is that *s* be bounded.
r. Ralph would not have attended the party unless he had considered himself obliged to attend.
s. Ralph didn't attend the party unless he either felt well or considered himself obliged to attend.
t. The conference will occur in Goldwin-Smith Hall, a vine-covered building on the Arts quad.
u. A necessary and sufficient condition for the sheik to be happy is that he has wine, women, and song.
v. If Mr Jones is happy, Mrs Jones is not happy, and if Mr Jones is not happy, Mrs Jones is not happy.
w. Adam will only go to the party if Barbara goes and alcohol is served, and he won't come if Cindy is invited.
x. Either Sam will come to the party and Max will not, or Sam will not come to the party and Max will enjoy himself.
y. Although I wouldn't have gone to the party if I hadn't been invited, Adam would have gone whether or not he was invited.
z. Provided that the newspaper reports are correct and all members of the Green gang have been arrested, they will all be convicted if and only if the D.A. has neither been bribed nor threatened.

Formalizing English arguments

1. We can think of an English language argument as an ordered pair whose first component is a set of sentences (the premises of the argument) and whose second component is a sentence (the conclusion of the argument): $<\{P_1, \ldots, P_n\}, C>$.

2. Suppose that $<\{P_1, \ldots, P_n\}, C>$ is an English argument and that we can formalize the sentences $P_1, \ldots, P_n$, and C as the wffs $\varphi_1, \ldots, \varphi_n$ and ψ (respectively). Say that the semantic sequent '$\varphi_1, \ldots, \varphi_n \models \psi$' is a **formalization of the argument**.

3. An English argument is a **valid argument** just in case: any way the world might be which makes the premises all true also makes the conclusion true; or: there is no way the world might be which makes the premises all true and the conclusion false.

4. Result: Suppose that '$\varphi_1, \ldots, \varphi_n \models \psi$' is a formalization of the English argument $<\{P_1, \ldots, P_n\}, C>$. If '$\varphi_1, \ldots, \varphi_n \models \psi$' is a correct sequent then $<\{P_1, \ldots, P_n\}, C>$ is a valid argument. (But not conversely.)

   Proof. Suppose that the world is such a way that $P_1, \ldots, P_n$ are all true. Then $\varphi_1, \ldots, \varphi_n$ are all true, since they are formalizations of $P_1, \ldots, P_n$. But then ψ is true, because '$\varphi_1, \ldots, \varphi_n \models \psi$' is a correct sequent. But then C is true, because it can be formalized as ψ. So any way the world might be which makes $P_1, \ldots, P_n$ all true also makes C true. So $<\{P_1, \ldots, P_n\}, C>$ is a valid argument. ■

   So we have a way of proving that an English language argument is valid: formalize it as a sequent about PC, and then prove that the sequent is correct (by any one of the three methods that we have been using – directly, by using truth tables, or by using tableau).

5. The converse of this result is false: some formalizations of valid English arguments yield sequents that are not correct. So just because an argument can be formalized as a sequent that is not correct, we should not conclude that the argument is invalid.

   Example: 'Grass is green; therefore grass is green' (a valid argument) can be formalized as '$A \models B$' (an incorrect sequent).

6. Note also that some valid English arguments *cannot* be formalized as correct sequents about PC. So just because an argument *cannot* be formalized as a correct sequent about PC, we should not conclude that the argument is invalid.

   Example: 'All men are mortal; Socrates is a man; therefore Socrates is mortal.'

7. To prove that an argument is invalid we need to provide a counterexample – a way the world might be which makes the premises all true and the conclusion false.

8. Exercises

Formalize the following arguments as sequents about PC, and then test the sequent of correctness (use the sentence letters suggested):

a. Jones is happy and relaxed, therefore Jones is happy. (‘H’, ‘R’)
b. If $a = 0$ or $b = 0$, then $ab = 0$. But $ab \neq 0$. Hence, $a \neq 0$ and $b \neq 0$. (‘A’, ‘B’, ‘P’)
c. If Jones is a Communist, Jones is an atheist. Jones is an atheist. Hence, Jones is a Communist. (‘C’, ‘A’)
d. If the number *x* ends in ‘0’, it is divisible by 5. *x* does not end in ‘0’. Hence *x* is not divisible by 5. (‘Z’, ‘D’)
e. Either the manager didn’t notice the change or else he approves of it. He noticed it all right. So he must approve of it. (‘N’, ‘A’)
f. If either Socrates was happily married or else he wasn’t, then Socrates was a great philosopher. Therefore Socrates was a great philosopher. (‘H’, ‘G’)
g. If *x* is positive, then *y* is negative. If *z* is negative, then *y* is negative. Therefore, if *x* is positive or *z* is negative, then *y* is negative. (‘X’, ‘Y’, ‘Z’)
h. Jones will come if she gets the message, provided that she is still interested. Although she didn’t come, she is still interested. Therefore she didn’t get the message. (‘C’, ‘M’, ‘I’)
i. If the temperature and air pressure remained constant, there was no rain. The temperature did remain constant. Therefore, if there was rain, then the air pressure did not remain constant. (‘T’, ‘A’, ‘N’)
j. If the first disjunct of a disjunction is true, then the disjunction as a whole is true. Therefore, if both the first and second disjuncts of the disjunction are true, then the disjunction as a whole is true. (‘F’, ‘W’, ‘S’)
k. If Forbush wins the election, then taxes will increase if the deficit will remain high. If Forbush wins the election, the deficit will remain high. Therefore, if Forbush wins the election, taxes will increase. (‘F’, ‘T’, ‘D’)
l. Either the robber came in the door, or else the crime was an inside one and one of the servants is implicated. The robber could come in the door only if the latch had been raised from the inside; but one of the servants is surely implicated if the latch was raised from the inside. Therefore one of the servants is implicated. (‘D’, ‘I’, ‘S’, ‘L’)
m. If capital investment remains constant, then government spending will increase or unemployment will result. If government spending will not increase, taxes can be reduced. If taxes can be reduced and capital investment remains constant, then unemployment will not result. Hence, government spending will increase. (‘I’, ‘S’, ‘U’, ‘T’)
n. If fallout shelters are built, other countries will feel endangered and our people will get a false sense of security. If other countries will feel endangered, they may start a preventive war. If our people will get a false sense of security, they will put less effort into preserving peace. If fallout shelters are not built, we run the risk of tremendous losses in the event of war. Hence, either other countries may start a preventive war and our people will put less effort into preserving peace, or we run the risk of tremendous losses in the event of war. (‘B’, ‘E’, ‘F’, ‘W’, ‘L’, ‘T’)

<u>The limitations of PC</u>

1. PC has a certain amount of **expressibility**. We can use it to disambiguate the sentence ‘Albert kissed Sue and Bert thanked Sue or Chris spoke to Sue’, and we can use it to prove the validity of the argument ‘Albert kissed Sue and Bert thanked Sue, therefore Albert kissed Sue’.

2. But it has limited expressibility. We cannot use it to disambiguate ‘Everyone kissed someone’, and we cannot use it to prove the validity of the following arguments:

   a. Albert kissed Sue; therefore someone kissed Sue.
   b. All men are mortal; Socrates is a man; therefore, Socrates is mortal.
   c. Any friend of Martin is a friend of John; Peter is not John’s friend; hence, Peter is not Martin’s friend.
   d. All human beings are rational; some animals are human beings; hence, some animals are rational.

3. In the case of the arguments, this is because their validity is not due to the meanings of the words ‘not’, ‘and’, ‘or’, ‘if’, or ‘if and only if’. Rather, they are due to the meanings of the words ‘all’, ‘any’, ‘every’, and ‘some’.

4. We will now extend our language to capture the meanings of these words as well – to a language with greater expressive resources, QC.

5. We will formalize the above arguments as the following correct sequents about QC:

   a. ‘Kas’ $\models$ ‘$\exists x \mathrm{K}x\mathrm{s}$’
   b. ‘$\forall x(\mathrm{M}x \rightarrow \mathrm{D}x)$’, ‘$\mathrm{Ms}$’ $\models$ ‘Ds’
   c. ‘$\forall x(\mathrm{Fm}x \rightarrow \mathrm{Fj}x)$’, ‘$\neg\mathrm{Fjp}$’ $\models$ ‘$\neg\mathrm{Fmp}$’
   d. ‘$\forall x(\mathrm{H}x \rightarrow \mathrm{R}x)$’, ‘$\exists x(\mathrm{A}x \wedge \mathrm{H}x)$’ $\models$ ‘$\exists x(\mathrm{A}x \wedge \mathrm{R}x)$’