

Extensions and intensions

1. We've been looking at the following approach to language: sentences have truth conditions; their truth conditions are determined by some semantic feature of their parts (called their *denotation*) and the manner in which they are composed.
2. We can define the *extension* of an expression:
  - a. The extension of a singular term is the *particular* to which it refers. So the extension of 'London' is London, and the extension of 'The best town in Australia' is Wagga Wagga. Note: (a) some singular terms have no extension, (b) perhaps we should relativise to a use (essential with 'I', 'he', 'that', etc.).
  - b. The extension of a predicate is the *set of particulars* of which it is true. So the extension of 'is red' is  $\{x: \text{'is red' is true of } x\}$ ; that is,  $\{x: x \text{ is red}\}$ . Are there predicates with no extension?
  - c. The extension of a sentence is its *truth value*. So the extension of 'Ithaca is gorges' is True. Are there sentences with no extension? Perhaps we need to relativise to a structure: 'John saw Mary wearing panties.'
3. We can often take the denotation of an expression to be its extension: 'John is tall'. Swapping 'John' for any other expression with the same extension will not change the truth conditions; same with 'is tall'. Also 'Grass is green and snow is white'.

We sometimes say that 'John is tall' is *extensional* in 'John', and extensional in 'is tall'.

4. Sometimes the extension is not good enough. 'John wants a pet with a kidney', 'John wants a pet with a heart' – these have different truth values, even though 'pet with a kidney' and 'pet with a heart' have the same extension.

Another: 'It is necessary that  $2 + 2 = 4$ ' and 'It is necessary that grass is green'.

5. So sometimes we take the denotation of an expression to be its *intension*.
6. What is the intension of an expression? First, we relativise extensions to possible worlds:
  - a. The extension of a singular term at a world  $w$  is the particular to which it refers at  $w$ . So the extension of 'London' at a world  $w$  is London (if names are (strongly) rigid designators)), and the extension of 'the best town in Australia' is the best town in Australia in  $w$ .

- b. The extension of a predicate at a world  $w$  is the set of particulars of which it is true at  $w$ .
- c. The extension of a sentence at a world  $w$  is its truth value at  $w$ .

Note:

- The expression is used in the actual world, not in  $w$ .
  - An expression may have an extension at some worlds but not at others.
  - This assume no particular view about what possible worlds are – it just assumes that there are such things.
7. Then we can define the *intension* of an expression  $e$  to be the (a?) function  $f$  from possible worlds to extensions such that  $f(w)$  is the extension of  $e$  at  $w$ .
  8. The intension of a sentence is a function from possible worlds to truth values. Such a function determines a set of possible worlds: the set of worlds at which the function takes the value True, or the set of worlds at which the sentence is true. Sometimes the function and the set are taken to be the same thing.
  9. Propositions are often taken to be either functions from possible worlds to truth values, or sets of possible worlds. In that case, the intension of a sentence is a proposition (or at least determines one).
  10. Can we take the *meaning* of an expression to be its intension?
    - a. They are more fine-grained: ‘is renate’ and ‘is cordate’ have different intensions, as do ‘grass is green’ and ‘snow is white’.
    - b. And we can show how the intension of ‘It is necessary that S’ is determined by the intension of S: ‘It is necessary that S’ is true at world  $w$  iff S is true at all worlds. (Note: do we take the meaning of ‘It is necessary that’ to be an intension, or a function from intensions to intensions?)
    - c. But ‘is a triangle’ and ‘is a trilateral’ have the same intension, as do ‘ $2 + 2 = 4$ ’ and ‘ $3 + 5 = 8$ ’, so maybe intensions are still too coarse-grained.
 

This suggests that *structured* intensions might be better candidates for meanings.
    - d. We still have the problem of empty names, and of corefering names.

### Stalnaker on Propositional Concepts

1. When I utter ‘Grass is green’ I express a proposition. That proposition determines a function from possible worlds to truth values, a function that is true at a world just in case grass is green in that world. Take this function to *be* the proposition. We can represent

this proposition using a matrix. Suppose there are just three worlds,  $i$ ,  $j$  and  $k$ . Suppose that grass is green in  $i$  but not  $j$  or  $k$ . Then:

‘Grass is green’			
	$i$	$j$	$k$
	T	F	F

2. Which proposition I express does not depend on which world I am in when I utter the sentence (actually, it depends on  $i$ ,  $j$ , and  $k$  – more about this later):

‘Grass is green’			
	$i$	$j$	$k$
$i$	T	F	F
$j$	T	F	F
$k$	T	F	F

If my utterance is true then I am not in worlds  $j$  or  $k$ .

3. For some sentences, which proposition I express does depend on the world of utterance. Take the sentence ‘You are a fool’. Let  $i$  be a world in which I am talking to O’Leary, O’Leary is a fool, and Daniels is not a fool. Let  $j$  be a world in which I am talking to O’Leary, O’Leary is not a fool, and Daniels is a fool. Let  $k$  be a world in which I am talking to Daniels, O’Leary is a fool, and Daniels is not a fool. Then:

‘You are a fool’			
	$i$	$j$	$k$
$i$	T	F	T
$j$	T	F	T
$k$	F	T	F

This illustrates that there are two independent reasons why two people might disagree about the truth value of an utterance.

4. Every sentence determines such a matrix. It represents what Stalnaker calls a *propositional concept*: a function from possible worlds to propositions, or (equivalently) a function from ordered pairs of possible worlds to truth values. This is sometimes taken to be the meaning of the sentence.
5. Think of the diagonal as representing a proposition. Call this the *diagonal proposition* of the sentence.
6. In some cases, the diagonal proposition of a sentence is necessarily true (i.e. true in every world). Suppose that  $i$  is a world in which I am in Ithaca,  $j$  is a world in which I am in Oxford, and  $k$  is a world in which I am in Wagga Wagga. Then:

‘I am here’

	<i>i</i>	<i>j</i>	<i>k</i>
<i>i</i>	T	F	F
<i>j</i>	F	T	F
<i>k</i>	F	F	T

An utterance of ‘I am here’ is true in every context, even though it expresses a contingent proposition in every context – an example of the *contingent a priori*.

7. An example of the *necessary a posteriori*. Let *i* be the actual world; let *j* and *k* be worlds in which the planet that is seen in the morning is distinct from the planet that is seen in the evening. Then:

‘Hesperus is Phosphorus’

	<i>i</i>	<i>j</i>	<i>k</i>
<i>i</i>	T	T	T
<i>j</i>	F	F	F
<i>k</i>	F	F	F

8. We can think of ‘It is not the case that’ and ‘It is necessary that’ and ‘An utterance of ‘...’ is true’ as being *sentence operators*. They apply to a sentence to yield a sentence. They determine functions from propositional concepts to propositional concepts. How do they effect the propositional concepts of the sentences they operate on? Suppose that in *i* I am in Ithaca, in *j* I am in Oxford, and in *k* I am in Wagga:

‘I am here’

	<i>i</i>	<i>j</i>	<i>k</i>
<i>i</i>	T	F	F
<i>j</i>	F	T	F
<i>k</i>	F	F	T

‘It is not the case that I am here’ (an extensional operator)

	<i>i</i>	<i>j</i>	<i>K</i>
<i>i</i>	F	T	T
<i>j</i>	T	F	T
<i>k</i>	T	T	F

‘It is necessary that I am here’ (a one-dimensional operator)

	<i>i</i>	<i>J</i>	<i>k</i>
<i>i</i>	F	F	F
<i>j</i>	F	F	F
<i>k</i>	F	F	F

‘An utterance of ‘I am here’ is true’ (a two-dimensional operator)

	<i>i</i>	<i>j</i>	<i>k</i>
<i>i</i>	T	T	T
<i>j</i>	T	T	T
<i>k</i>	T	T	T

9. Now suppose that in  $i$  I am talking to O’Leary, O’Leary is a fool, and Daniels is not a fool; in  $j$  I am talking to O’Leary, O’Leary is not a fool, and Daniels is a fool; in  $k$  I am talking to Daniels, O’Leary is a fool, and Daniels is not a fool:

‘You are a fool’

	$i$	$j$	$k$
$i$	T	F	T
$j$	T	F	T
$k$	F	T	F

‘An utterance of ‘You are a fool’ is true’ (two-dimensional operator)

	$i$	$j$	$k$
$i$	T	F	F
$j$	T	F	F
$k$	T	F	F

10. Stalnaker’s treatment of the negative existential, ‘Sherlock Holmes does not exist’. Let  $i$  be the actual world; let  $j$  be a world in which a famous detective named ‘Sherlock Holmes’ lived in London and Doyle wrote a series of historical accounts of his cases; and let  $k$  be a world in which Doyle was a famous detective named ‘Sherlock Holmes’ and wrote about himself under the name ‘Doyle’. Then:

‘Sherlock Holmes does not exist’

	$i$	$j$	$k$
$i$	T	T	T
$j$	T	F	T
$k$	F	F	F