Lecture 9.2

Quantification

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1. In the previous lecture we looked at a compositional semantics for sentences of the form '*X Y*', where '*X*' is replaced by a singular definite NP, and '*Y*' is replaced by a VP. It this lecture we will extend that theory.

2. **First extension:** to the fragment of English that consists of all sentences of the form '*X V Z*', where '*X*' and '*Z*' are replaced by singular definite NPs, and '*V*' is replaced by a transitive verb (such as 'John loves Mary'). The theory is this:

   - The denotation of the NP replacing '*X*' is still an object. (In particular, the denotation of 'John' is John.)
   - The denotation of the transitive verb replacing '*V*' is a relation, i.e. a set of ordered pairs. In particular, the denotation of 'loves' is the set of ordered pairs <x, y> such that x loves y.
   - The denotation of the NP replacing '*Z*' is an object. (In particular, the denotation of 'Mary' is Mary.)
   - The denotation of a VP of the form '*V Z*' is the set of objects which stand in the relation denoted by the verb that replaces '*V*' to the object denoted by the NP that replaces '*Z*'. E.g., the denotation of 'loves Mary' is the set of objects that love Mary.
   - If S is a sentence of the form '*X V Z*' then S is true iff the denotation of the noun phrase that replaces '*X*' is a member of the denotation of the verb phrase that replaces '*V Z*'.

   According to this theory, 'John loves Mary' is true iff the denotation of 'John' is a member of the denotation of 'loves Mary'; that is, iff John is a member of the set of objects that love Mary; that is, iff John loves Mary. So, according to this theory, 'John loves Mary' is true iff John loves Mary.

3. **Second extension: quantified subjects**. The following sentences each have a quantifier as their subject. How are their meanings determined compositionally?
   - All dogs have hair
   - Some dogs have hair
   - No dogs have hair
   - Most dogs have hair
   - The dog has hair

4. A first attempt: make no change to the theory. That is, the quantifier (eg. 'All dogs') denotes an object, the verb phrase (eg. 'has hair') denotes a set of objects, and the sentence is true iff the object denoted by the quantifier is an element of the set denoted by the verb phrase.

   That might be ok for the last example, but it is problematic for the first four.

5. A better theory: still say that the verb phrase denotes a set of objects, but say instead that the quantifier denotes a <u>set of sets</u> of objects; the sentence is true iff the set denoted by the verb phrase is an element of the set denoted by the quantifier.

   Which set of sets does each quantifier denote?

- ‘All dogs’ denotes the set of sets that contain all dogs
- ‘Some dogs’ denotes the set of sets that contain some dogs
- ‘No dogs’ denotes …
- ‘Most dogs’ denotes …
- ‘The dog’ denotes …
- ‘Exactly two dogs’ denotes …

This allows us to derive the right truth conditions. Eg. ‘All dogs have hair’ is true iff the set denoted by ‘has hair’ is an element of the set denoted by ‘All dogs’; that is, iff the set of objects that have hair is an element of the set of sets that contain all dogs; that is, iff the set of objects that have hair contains all dogs; that is, iff all dogs have hair. So ‘All dogs have hair’ is true iff all dogs have hair.

6. But what about the quantifiers? They have structure: each consists of a determiner followed by a noun phrase. How are *their* meanings determined compositionally?

   We can say this: the noun phrase denotes a set of objects; the determiner denotes a function which takes a set and produces a set of sets; the quantifier denotes the value of the function applied to the set.
   - ‘dogs’ denotes the set of dogs
   - ‘All’ denotes the function which takes a set S and produces the set of sets that contain every element of S. So ‘All dogs’ denotes the set of sets that contain all dogs.
   - ‘Some’ denotes the function which takes a set S and produces the set of sets that contain some element of S. So ‘Some dogs’ denotes the set of sets that contain some dogs.

   We then have a compositional semantics for sentences of the form ‘$\mathcal{D}\ \mathcal{A}\ \mathcal{B}$’

7. We can think of this another way. Each determiner denotes <u>a relation between sets</u>. A sentence of the form ‘$\mathcal{D}\ \mathcal{A}\ \mathcal{B}$’ is true iff the sets denoted by ‘$\mathcal{A}$’ and ‘$\mathcal{B}$’ stand in the relation denoted by ‘$\mathcal{D}$’:
   - ‘All A B’ is true iff $A \subseteq B$
   - ‘Some A B’ is true iff $A \cap B \neq \varnothing$
   - ‘No A B’ is true iff $A \cap B = \varnothing$
   - ‘Most A B’ is true iff …
   - ‘The A B’ is true iff …
   - ‘At least three A B’ is true iff …
   - ‘Exactly three A B’ is true iff …

8. Sometimes we need to think of names as denoting sets of sets as well: ‘John and all his girlfriends came to the party’. Which set of sets does ‘John’ denote?

9. **Next problem: quantified objects**.
   - ‘John loves every woman’